Identification of Aircraft Dynamics Using Hammerstein-Wiener Nonlinear Model

A. Roudbari
Aerospace Engineering
Sharif University of Technology
Tehran, Iran
alirezaroudbari@ae.sharif.edu

F.Saghafi
Aerospace Engineering
Sharif University of Technology
Tehran, Iran
saghafi@sharif.edu

***Abstract*— In this article, a new approach based on block-oriented nonlinear models for modeling and identification of aircraft nonlinear dynamics has been proposed. Some of the block-oriented nonlinear models are considered as flexible structures which are suitable for the identification of widely applicable dynamic systems. These models are able to approximate a wide range of system dynamics. Flying vehicle are such nonlinear systems whose dynamics depend not only on pilot control inputs but also on flight conditions, i.e., Mach and altitude. In this study, three systems of Hammerstein, Wiener, and Hammerstein-Wiener for identification and modeling of aircraft nonlinear dynamics have been used and compared. The results of the study show that these models are able to identify and model aircraft nonlinear dynamics. In order to compare each model's performance, the criteria of the percentage of best fit, final prediction error, and loss function for training data and validation have been considered. The results show that, Hammerstein-Wiener system has a better performance in extracting flexible black-box model based on experimental data from aircraft flight test data.**

Keywords*— Aircraft nonlinear dynamics, System identification, Hammerstein- Wiener modle, flight test.

I. Introduction

Modeling and simulation are widely used as essential tools to predict and analyze complex systems in various scientific and engineering fields. For an aerospace system such as aircraft, mathematical models are useful for dynamic analysis, flight simulation, guidance and navigation studies, controller design and validation, air combat investigations, pilot training, and many other tasks. In general, the aircraft flight dynamics is a nonlinear and coupled system whose dynamic modeling, in addition to pilot control inputs, depends on flight conditions, i.e., Mach and altitude. When there is a need for a wide variety of system domains, linear hypothesis loses its validity and nonlinear models for predicting system behavior become necessary. Modeling these systems with linear structures requires simpler hypotheses which reduce accuracy. While dealing with nonlinear and complicated systems, system identification is an appropriate and common solution in modeling. The purpose of system identification is to obtain the most suitable mathematical model or transfer function from the real system based on main system's input-output data. Generally, structures of the identification algorithms are classified as linear and nonlinear. Over the past few decades, classic linear identification methods have developed to such an extent that now they are considered as standard tools in a variety of engineering fields to solve most problems related to linear identification. With respect to the fact that the linear models are only valid in a small domain around operational points and most of the systems are actually nonlinear, nonlinear mapping will be necessary for high fidelity modeling tasks.

In system identification, choosing model structure is of utmost importance and sometimes it is convenient to use a flexible model structure capable of approximating a wide variety of systems behavior. One of the main research areas in nonlinear modeling is block-oriented nonlinear models [1-2]. Hammerstein (H) [3], Wiener (W) [4], and the combination of these two models called Hammerstein-Wiener (HW) are three general models of block-oriented nonlinear models. These are the simplest of dynamic block-oriented nonlinear systems which include internal connection from linear time independent linear system static nonlinear dynamics. So far, different methods have been suggested for the identification of these models' parameters. However, very few, if any, studies have been conducted on nonlinear systems identification using block-oriented parametric models [4-7]. In this study, using the data obtained from flight tests and with the development of Hammerstein-Wiener model, the six-degree-of-freedom nonlinear dynamics is identified, in a way that the obtained model is able to approximate the aircraft dynamic behavior with acceptable accuracy without the main system. This identification method is only based on input and output data obtained from flight tests.

In order to validate more, the reported results in this study have been compared with the obtained data from a 4th generation fighter aircraft and helicopter. The three systems have been used in the forms of single input single output (SISO), multi input single output (MISO), and multi input multi output (MIMO) for the identification of aircraft dynamics.

The comparison of the obtained data from the training data and model validating data shows the superiority of HW model over Hammerstein model.

II. Block-Oriented Nonlinear Model Structure

One of the most important steps in the identification of a nonlinear system is choosing a nonlinear model structure in order to show the system being tested. H and W models are the simplest dynamic block-oriented nonlinear systems which, due to their simplicity and having a physical concept, are used in a variety of domains. Different nonlinear systems along with different nonlinear elements require different structures of H and W. Serial combination of H and W models will create a new model called HW. These new structures have great capabilities in modeling. It is clear that HW model performs better with nonlinear functions. So far, a host of methods have been proposed for their identification.

A. Hammerstein model

Hammerstein (H) is the combination of a static nonlinear block and a linear dynamic block (Figure 1). In this structure, static nonlinear functions offer the possibility of wider domains of system dynamics compared to fully linear models. In this model, the input u(t) first enters nonlinear function f, then the output of this function, when entering the nonlinear block G_2 as internal signal w(t), is filtered. This system has come to be known as nonlinear-linear (N-L). There are several algorithms for the identification of H model [8-10]. More accurately, for each scalar quantity for an input string u(t) and the related output y(t), this model is shown as an operator $\Gamma(\beta)$ in u(t) as the following.

$$y(t)=\Gamma(\beta)u(t) \tag{1}$$

Parameters and variables in this equation are defined as follows:

$$\beta=[\eta,\theta] \tag{2}$$

$$w_j(t)=G_j(q^{-1},\theta)u_j(t) \tag{3}$$

$$y(t)=G(q^{-1},\theta)w(t)+e(t) \tag{4}$$

Transfer function $G(q^{-1},\theta)$ is defined as follows:

$$G_j(q,\theta_j)=\frac{B_{ij}(q,\theta)}{F_{ij}(q,\theta)} \tag{5}$$

$$B_j(q,\theta)=b_0q^{-1}+b_1q^{-2}+\dots b_{nb}q^{-n} \tag{6}$$

$$F_j(q,\theta)=f_0q^{-1}+f_1q^{-2}+\dots f_{nf}q^{-n} \tag{7}$$

$$\theta=[b_0 \quad b_1 \quad \dots \quad b_{nb} \quad f_0 \quad f_1 \quad \dots \quad f_{nf}] \tag{8}$$

$i=1,2,\dots n_y$ and $j=1,2,\dots n_u$

$u_j(t)$ ، $w_j(t)$ are the inputs and outputs respectively, $B_j(q,\theta)$ و $F_j(q,\theta)$ are numerically polynomial in delay operator unit, q^{-1} و θ are parameters vector with the coefficients of the polynomial linear transfer function.

Function $f_1(*,\eta)$ is a nonlinear function without memory, which the only restriction placed on it is that its derivatives exist for all the elements.

Static nonlinear transfer function $f_j(w_j(t),\eta)$ can be chosen as different functions such as stepwise linear function, single-layer neural network, wavelet function, saturation function, dead zone function, and one-dimensional polynomial function. If a polynomial function from the order of n_j is used, model output will be as follows:

$$w_j(t)=\sum_{k=1}^{n_j}\alpha_{jk}u_j^k(t) \tag{9}$$

By using transfer function $G(q^{-1},\theta)$ with the order n, the output of the system is defined as follows

$$\hat{y}(t)=G_j(q^{-1},\theta_j)w_j(t)=\frac{B_{ij}(q,\theta)}{F_{ij}(q,\theta)}w_j(t) \tag{10}$$

$$y(t)=\sum_{j=1}^{m}\hat{y}(t)+E(t)=$$
$$\sum_{j=1}^{m}\left[\sum_{k=1}^{m_j}b_{j0}\alpha_{jk}.u_j^k(t)+\frac{B^*{}_j(q^{-1},\theta)}{F_j(q^{-1},\theta)}w_j(t)\right]+\frac{C(q^{-1},\theta)}{D(q^{-1},\theta)}e(t) \tag{11}$$

Which

$$B_j^*(q^{-1},\theta)=b_{j1}^*q^{-1}+b_{j2}^*q^{-2}+\dots b_{jn_j}^*q^{-n} \tag{12}$$

$$b_{jk}^*=b_{jk}-b_{j0}.f_{jk} \tag{13}$$

B. Wiener model

With studying Wiener (W) theory, it can be concluded that each system from the W class can be shown as a model in Figure. 2. this model consists of a linear and nonlinear dynamic blocks in a series manner. As shown in Figure 3, the input signal u(t) is first applied to the filter block G_1 and then the output block G_1 is defined with a static nonlinear function *f*. Dynamic linear filter output has been shown by w(t). This model is known as linear-nonlinear (L-N). In this structure, for each scalar quantity for an input string u(t) and the related output y(t), W model is shown as an operator $\Gamma(\beta)$ in u(t) as the following.

$$y(t)=\Gamma(\beta)u(t) \tag{14}$$

Parameters and variables in this equation are defined as:

$$\beta=[\eta,\theta] \tag{15}$$

$$w_j(t)=\sum_{k=1}^{n_j}\alpha_{jk}u_j^k(t) \tag{16}$$

Transfer function $G(q^{-1},\theta)$ is defined as the following:

$$G_j(q,\theta_j)=\frac{B_{ij}(q,\theta)}{F_{ij}(q,\theta)} \tag{17}$$

$$B_j(q,\theta)=b_{j0}q^{-1}+b_{j1}q^{-2}+\dots b_{jn_b}q^{-n_b} \tag{18}$$

$$F_j(q,\theta)=f_{j0}q^{-1}+f_{j1}q^{-2}+\dots f_{jn_f}q^{-n_f} \tag{19}$$

$$\theta=[b_0 \quad b_1 \quad \dots \quad b_{nb} \quad f_0 \quad f_1 \quad \dots \quad f_{nf}] \tag{20}$$

$i = 1,2,...n_y$ and $j = 1,2,...n_u$

In this system, static nonlinear transfer function $f_j(w_j(t),\eta)$ consists of different functions such as stepwise linear function, single-layer neural network, wavelet function, saturation function, dead zone function, and one-dimensional polynomial function. If a polynomial function from the order of n_j is used, model output will be as follows:

$$\hat{y}(t) = \sum_{k=1}^{n_j} \alpha_{jk} w_j^k(t) = \alpha_{j1} w_j(t) + \sum_{k=2}^{n_j} \alpha_{jk} w_j^k(t) \qquad (21)$$

where n_j is order polynomial. By using $w_j(t)$ and transfer function $G_j(q^{-1},\theta)$ in the equation we will have:

$$y(t) = \sum_{j=1}^{m} \hat{y}(t) + E(t) =$$
$$\sum_{j=1}^{m} \left[\frac{\alpha_{j1}.B_j(q^{-1},\theta)}{F_j(q^{-1},\theta)} + \sum_{k=2}^{n} \alpha_{jk} w_j^k(t) \right] + \frac{C(q^{-1},\theta)}{D(q^{-1},\theta)} \varepsilon(t) \qquad (22)$$

A series combination of H and W models will create a new structure called H-W.

C. Hammerstein- Wiener system

Hammerstein-Wiener model (HW) is the combination of H and W models. This model is the result of connecting a series of input nonlinear static block (f_1) with a dynamic linear system (G_1) (Linear Static) which is followed by an output nonlinear static block (f_2) (Figure 3). Thus, this system is called a 'Sandwich' or an nonlinear-linear-nonlinear (N-L-N). *u(t)* and *y(t)* are input and output signals respectively; and, $w_1(t)$ and $w_2(t)$ are internal signals. The internal signals are not available for measurement. Signal $w_1(t)$ passes through a linear transfer function (G_1) and its dimensions are the same as those of y(t) output. The first nonlinear static block f_1 can be explained as:

$$w_1(t) = f_1(u(t),\eta) \qquad (23)$$

$w_1(t)$ and u(t) are the inputs and outputs of the first block respectively. Output linear dynamical system can be written as:

$$w_2(t) = \frac{B_{ji}(q^{-1},\theta)}{F_{ji}(q^{-1},\theta)} w_1(t) = b_{10} w_1(t) + \frac{B_1^*(q^{-1},\theta)}{F_1(q^{-1},\theta)} w_1(t) \qquad (24)$$

$$B_j(q,\theta) = b_{j0} q^{-1} + b_{j1} q^{-2} + ... b_{jn_b} q^{-n} \qquad (25)$$

$$F_j(q,\theta) = f_{j0} q^{-1} + f_{j1} q^{-2} + ... f_{jn_f} q^{-n} \qquad (26)$$

$$\theta = [b_0 \quad b_1 \quad ... \quad b_{nb} \quad f_0 \quad f_1 \quad ... \quad f_{nf}] \qquad (27)$$

$$b_{1k}^* = b_{1k} - b_{10}.f_{1k} \qquad (28)$$

where $j = 1,2,...n_y$ and $i = 1,2,..n_u$

The second nonlinear static block $f_2(w_2(t),\eta)$ can be explained as:

$$y(t) = f_2(w_2(t),\eta) \qquad (29)$$

Linear transfer function has the inputs $w_2(t)$ and outputs $y(t)$. The inputs u(t) and outputs $y(t)$ of HW model are able to be measured, while the internal variables of $w_1(t)$ and $w_2(t)$ cannot be measured. If there are two continuous nonlinear functions in HW system, polynomial of orders m_1 and m_2 can be used for the first and second nonlinear blocks:

$$w_1(t) = \sum_{k=1}^{m} \alpha_{1k} u^k(t) \qquad (30)$$

$$\hat{y}(t) = \sum_{k=1}^{m_2} \alpha_{2k} w_2^k(t) = \alpha_{21} w_2(t) + \sum_{k=2}^{m_2} \alpha_{2k} w_2^k(t) \qquad (31)$$

$$y(t) = \sum_{k=1}^{m} \hat{y}(t) + E(t) =$$
$$\sum_{k=1}^{m_1} \alpha_{1k} u^k(t) + \frac{B_1(q^1,\theta)}{F_1(q^{-1},\theta)} w_1(t) + \sum_{k=2}^{m_2} \alpha_{2k} w_2^k(t) + \frac{C(q^{-1},\theta)}{D(q^{-1},\theta)} e(t) \qquad (32)$$

W and H systems are specific cases of this model. These systems are advantageous for the control which is based on different nonlinear process prediction models in industry [11]. For example, the first nonlinear block can function as actuator. This model with different structures can be used for identification system modeling.

III. DATA GENERATION

One important step in system identification is the extraction of suitable aircraft flight data. The obtained data should include comprehensive information on the aircraft dynamics; therefore, it is necessary to define and apply suitable inputs to the aircraft. These inputs are applied by three levels of control including elevator, rudder, and aileron. The recorded flight data used for the identification include longitudinal acceleration (n_x), latitudinal acceleration (n_y), vertical acceleration (n_z), position angles of pitch (θ), Roll (φ), angle of yaw (ψ), angle of attack (AOA), and velocity (v). The type of input signal for the aircraft is important in order to collect identification data in the process of learning and validation of the identified model. In this study, multi-step and sinusoidal inputs have separately been used for each controller. The experimental data from the helicopter are obtained from *XV-15* helicopter flight tests. Frequency sweep input has been used for the outputs in the flight tests.

IV. SIMULATION RESULTS

The three structures of H, W, and HW with different linear and nonlinear functions have been used for the identification of aircraft dynamics. Input and output data derived from flight recorder device for system identification are divided into two categories of training and test or validation. Figure 5 shows the training and validation data for the AOA variable with elevator control input. Given that in these structures, nonlinear functions and linear transfer functions can be used with a variety of orders; different structures for the identification of aircraft dynamics have come into existence. Having applied different nonlinear functions in the input and output blocks of the three systems, it was found that sigmoid nonlinear function has the best approximation and the fewest errors in identification and modeling of different aircraft data. Thus, sigmoid function was used for the three systems. Choosing the order of the linear transfer function in different blocks plays an important role in reducing the output error of the identified model. Tables 1 to 4 show the percentage of best fit, final prediction error (FPE), and loss function (LF) for the training data and test. The best fit for the data is calculated through the following:

$$FIT = \left(1 - \frac{\sqrt{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}}{\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})^2}}\right).100 \tag{33}$$

where y_i is the measured output, $\hat{y}$ is the approximated or simulated output, $\bar{y}$ is the average of approximated output based on the best fit, and N is the number of sampling data. The more this value is for training data and validation, the better the performance of the obtained model. FPE criterion is also used for the evaluation of the identified model. Based on Akaike theory [4], the model with lower FPE is more accurate. FPE is calculated as:

$$FPE = lf.\left(1 + \frac{2d}{N}\right) \tag{34}$$

where lf is the loss function, d is the number of parameters to be approximated, and N is the number of sampling data. Loss function is calculated as:

$$lf = \det\left(\frac{1}{N}\sum_{1}^{N}\varepsilon(t,\theta_N)(\varepsilon(t,\theta_N))^T\right) \tag{35}$$

where θ_N is the approximated parameter and ε is the model output error. The criterion for the selection of transfer function order for each block is based on the highest fit, the lowest final prediction error, and the function loss for the test data and training. By comparing these criteria for each one of the aircraft variables, it is clear that by increasing the order of linear transfer function, the output performance of the identified model improves; but, for validation data, by increasing the order of linear function it is possible that the output error model will increase as well. There is no suitable criterion for the selection of linear function order in each of the models. The results of the best state for different aircraft parameters using HW model are:

Angel of attack:

$$w_1(t) = \frac{1}{e^{-u(t)} + 1} \tag{36}$$

$$w_2(t) = \frac{B(q^{-1},\theta)}{F(q^{-1},\theta)} w_1(t)$$

$$B(q^{-1},\theta) = -0.01116q^{-1} - 0.04331q^{-2} \tag{37}$$

$$F(q^{-1},\theta) = 1 - 0.1551q^{-1} + 0.1126q^{-2} + 0.01834q^{-3} + 0.07852q^{-4} - 0.02967q^{-5}$$

$$y(t) = \frac{1}{e^{-w_2(t)} + 1} \tag{38}$$

Vertical acceleration:

$$B(q^{-1},\theta) = -0.00683q^{-1} - 0.0050q^{-2}$$
$$F(q^{-1},\theta) = 1 - 0.4835q^{-1} + 0.2432q^{-2} \tag{39}$$

Forward acceleration:

$$B(q^{-1},\theta) = -0.001347q^{-1} - 0.006422q^{-2}$$
$$F(q^{-1},\theta) = 1 - 0.1046q^{-1} + 0.1057q^{-2} + 0.0287q^{-3} \tag{40}$$

Lateral acceleration:

$$B(q^{-1},\theta) = 0.00001q^{-1} - 0.0001474q^{-3}$$
$$F(q^{-1},\theta) = 1 - 0.8719q^{-1} - 0.1779q^{-2} + 0.1128q^{-3} - 0.06274q^{-4} \tag{41}$$

In order to compare the performance of the three models, state variable of φ from a helicopter has been used. The results of the three models have been shown in Table 5 with the best identified model for each of the systems. By comparing this table, it has become evident that the performance of HW is better than the other two models. The output results of the three models have been shown in Figure 4 with the real data for the variable φ. For HW, models of second order for block filter G_1 have fewer errors for training data and test. But for HW model of fourth and fifth orders, output error model decreases to some extent. The results of simulation of this variable for the three models have been shown as:

Wiener model

$$w_1(t) = \frac{0.003561q^{-1} - 0.00356q^{-2}}{1 - 2q^{-1} + q^{-2}} u(t) \tag{42}$$

$$y(t) = \frac{1}{e^{-w_1(t)} + 1} \tag{43}$$

Hammerstein model

$$w_1(t) = \frac{1}{e^{-u(t)} + 1} \tag{44}$$

$$y(t) = \frac{-0.01062q^{-1} + 0.01061q^{-2}}{1 - 2q^{-1} + q^{-2}} w_1(t) \tag{45}$$

$$y(t) = \frac{1}{e^{-w_1(t)} + 1} \tag{46}$$

Hammerstein- Wiener model model

$$w_1(t) = \frac{1}{e^{-u(t)} + 1} \tag{47}$$

$$w_2(t)=\frac{B(q^{-1},\theta)}{F(q^{-1},\theta)}w_1(t)$$

$$B(q^{-1},\theta)=0.004568q^{-1}-0.004568q^{-2} \quad (48)$$

$$F(q^{-1},\theta)=1-2q^{-1}+q^{-2}$$

$$y(t)=\frac{1}{e^{-w_2(t)}+1} \quad (49)$$

Figure 6 shows the results of training for the aircraft's forward acceleration vector with three aircraft's control inputs. Figure 7 shows the results of validation with new inputs for the aircraft's forward acceleration vector. The results shown in these two figures represent that the identified model has approximated the aircraft's outputs well and is able to model the aircraft's dynamic behavior without the main system. In this figure, each aircraft state has been compared with the identified model's outputs. The results reported in these figures show that HW model is suitable for the identification of aircraft's nonlinear dynamics. Figure 8 shows the results of HW identification in the form of MISO. The results of training and validation show that this model has fewer errors compared to the previous model. Figure 9 shows the training results of HW model with multistep inputs. The results of training and validation with this input indicate that the identified model with this input has more errors compared to the situation in which the model has been identified with sinusoidal input. In the same way, the number of required data for the model's training is higher than that of the previous one.

V. CONCLUSION

In this article, by introducing a new approach to identification, block-oriented nonlinear models have been used for modeling and identification of aircraft nonlinear dynamics. In this study, three systems of Hammerstein, Wiener, and the combination of both called Hammerstein-Wiener, all of which are nonlinear by nature, have been used and compared for modeling and identification of aircraft's nonlinear dynamics. The procedure applied in this study is to build a relation between linear modeling techniques and the nonlinear approach introduced in this study for accurate modeling and identification of helicopter's and aircraft's complicated dynamics.

The obtained model is able to model highly maneuverable aircraft's six-degree-of-freedom dynamics. This structure has been used in three modes of SISO, MIMO, and MISO; of which, MISO has fewer errors in training and validation compared to the previous model. New structures having made up of the combination of H and W, compared to other methods, have higher capability and better performance in modeling aircraft dynamic behavior.

TABLE I. COMPARISON OF HAMMERSTEIN-WIENER MODEL STRUCTURE AND PERFORMANCE WITH DIFFERENT ORDERS FOR AOA

AOA	FIT on Train Data	FIT on Validation Data	FPE	Loss Function
nb=2 nf=1	76.95	70.57	0.2710	
nb=2 nf=2	77.94	72.71	0.2112	0.1591
nb=2 nf=3	78.43	73.59	0.2474	0.1350
nb=2 nf=4	79.26	71.20	0.2145	0.1361
nb=2 nf=5	79.91	73.22	0.2145	0.1357
nb=2 nf=6	7958	71.73	0.2085	0.1312

TABLE II. COMPARISON OF HAMMERSTEIN-WIENER MODEL STRUCTURE AND PERFORMANCE WITH DIFFERENT ORDERS FOR AIRCRAFT VERTIVAL ACCELERATION

n_y	FIT on Train Data	FIT on Validation Data	FPE	Loss Function
nb=2 nf=1	78.39	69.16	0.0222	0.0143
nb=2 nf=2	79.08	71.33	0.0210	0.0135
nb=2 nf=3	80.93	61.75	0.0175	0.0112
nb=2 nf=4	81.93	70.86	0.0159	0.0101
nb=2 nf=5	82.33	70.97	0.0153	0.0097
nb=2 nf=6	82.69	71.99	0.0147	0.0092

TABLE III. COMPARISON OF HAMMERSTEIN-WIENER MODEL STRUCTURE AND PERFORMANCE WITH DIFFERENT ORDERS FOR AIRCRAFT FORWARD ACCELERATION

n_x	FIT on Train Data	FIT on Validation Data	FPE	Loss Function
nb=2 nf=1	77.58	62.37	0.0279	0.0180
nb=2 nf=2	79.03	62.06	0.0241	0.0155
nb=2 nf=3	79.37	64.79	0.0236	0.0151
nb=2 nf=4	78.68	57.92	0.0249	0.0159
nb=2 nf=5	80.32	60.98	0.0225	0.0142
nb=2 nf=6	80.31	60.54	0.0227	0.0143

TABLE IV. COMPARISON OF HAMMERSTEIN-WIENER MODEL STRUCTURE AND PERFORMANCE WITH DIFFERENT ORDERS FOR AIRCRAFT LATERAL ACCELERATION

n_z	FIT on Train Data	FIT on Validation Data	FPE	Loss Function
nb=3 nf=1	83.74	78.24	0 .0228	0 .02286
nb=3 nf=2	84.99	67.02	0 .0002	0 .00021
nb=3 nf=3	82.28	16.34	0 .0003	0 .00032

nb=3 nf=4	90.32	79.81	0.00001	0.00001
nb=3 nf=5	91.16	68.48	0 .00028	0 .00028
nb=3 nf=6	86.35	76.38	0.000001	0.00001

TABLE V. COMPARISON OF DIFFERENT MODEL STRUCTURE AND PERFORMANCE WITH DIFFERENT ORDERS FOR HELICOPTER ROLL ANGLE RATE

φ	**FIT on Train Data**	**FIT on Validation Data**	**FPE**	**Loss Function**
W	84.11	87.62	3.86559	3.86308
H	89.34	86.78	2.45472	2.45313
HW	90.33	88.57	0.88852	0.88794

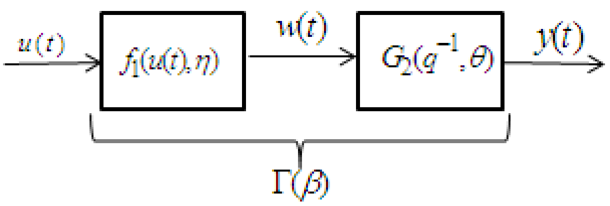

Fig.1. Hammerstein model

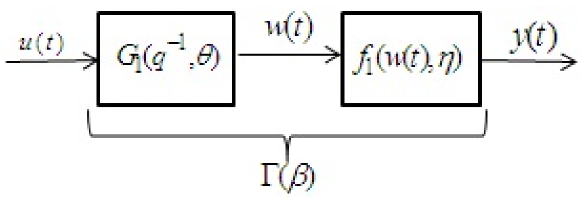

Fig.2. Wiener model

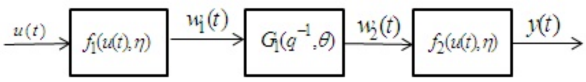

Fig.3. Hammerstein- Wiener model

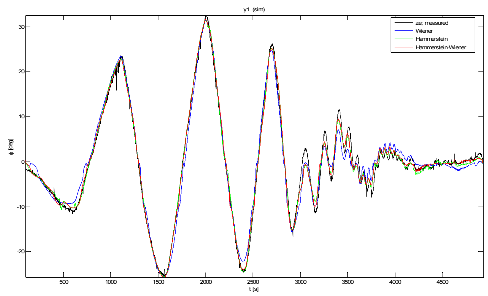

Fig. 4. Comparing tree models of Hammerstein, Wiener, and Hammerstein-Wiener output with helicopter roll angle data

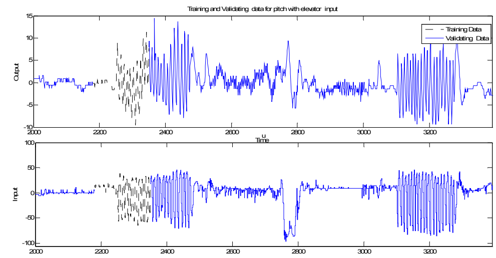

Fig. 5. Test data for training and validation

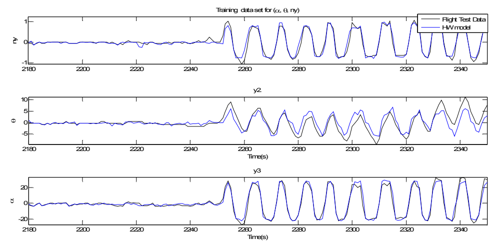

Fig. 6. Training results for HW model with sinusoidal input in MIMO

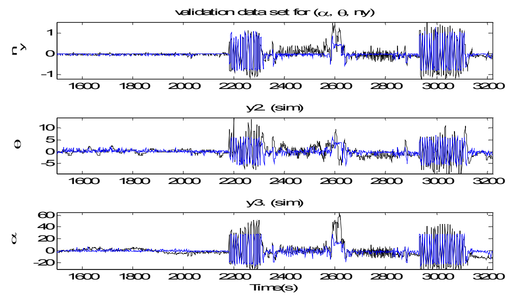

Fig. 7. Validation results for HW model with new input in MIMO

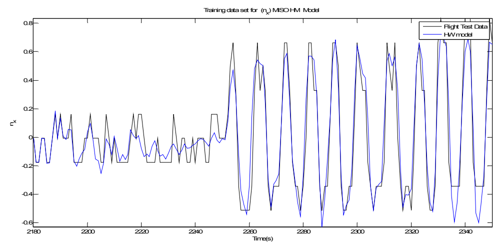

Fig. 8. Training results for HW model with sinusoidal input in MISO

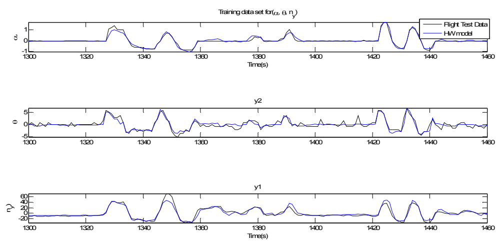

Fig.9. Training results for HW model with multistep input in MIMO

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