A Simplified Approach to Measure 21 Forms of Geometric Error for Three-axis Machine Tools: Principles and Application

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Abstract—The primary objective of this paper was the development of a simple method for testing 21 forms of geometric error associated with three-axis machine tools. To avoid Abbe's error, measurement devices must be placed along an ideal motion line when measuring geometric error. This results in a number of practical measurement difficulties, which necessitate the establishment a new measurement methods incorporating a geometric error model configured specifically to the location of the measurement device.

This paper introduces the principles and practical applications of a simple testing method using common measurement devices such as indicators or probes, gauge blocks, and straight gauges to measure 21 forms of geometric error. The results of this measurement can be used to provide error compensation for three-axis machine tools. We applied the traditional method of HTM to deduce a geometric error model for three-axis machine tools and simplified this model to a kinematic parameter-independent model. Finally, based on the new measurement method and compensation system corresponding to this error model, we established a truly simple and practical compensation technique providing increased accuracy for three-axis machine tools.

Keywords—three-axis machine tool; geometric error; HTM; error compensatio;

I. Introduction

Enhancing the accuracy of CNC machine tools is a crucial step in the development of this technology. Errors that detract from the accuracy of machine tools can be divided into three categories: structurally-induced errors, driver-induced errors, and quasi-static errors. According to the literature, quasi-static error, including both geometric and thermal error, accounts for 70 % of the error in CNC machine machining.

In 2008, a total volumetric compensation was introduced by Siemens in the 840D controller [1] and Heidenhain proposed the iTNC 530 in 2009 [2]. These functions increase the accuracy of machining, as long as volumetric errors were initially determined using suitable measurement technology.

This paper examined geometric error in quasi-static situations. The development of geometric error models for machine tools has been well developed over the past few years [3–7]. These models describe error in the position and orientation of tools relative to the workpiece in specific positions, whereby factors detracting from accuracy are the result of kinematic link parameters and individual sources of error. It is well understood that a lack of accuracy in the motion along a linearly driven axis is associated with six forms of motional error, including one form of linear error, two of straightness, and three rotational.

With modern measurement devices such as the 6D laser interferometer [8], all six forms of motional error can be measured rapidly. The LaserTRACER [9] offers an efficient, high-precision measurement system for volumetric calibration, but this measurement system is very expensive. The accuracy of three-axis machine tools can be dramatically improved through error compensation based on an error model [8,10~12].

Currently, geometric error modeling depends on a three-axis machine kinematic chain to create a geometric error model, the home position of which is regarded as the reference coordinate for motion along that axis. For this reason kinematic parameters between the coordinate systems of linear axes are needed to describe their relationship to motion. However, ideal axis lines and the center of revolution of the linear motion slide is difficult to define precisely making it impossible to define kinematic parameters. Furthermore, geometric error defined by the ideal axis line of linear motion slides must be measured by placing the measurement device on this axis line to avoid Abbe's error. This creates practical measurement difficulties when the linear motion slide is at a high position or when there is interference. Overall error is at the tool end of geometric error model with kinematic parameters constructed according to a machine reference coordinate system. In actual machining, however, a certain point on the workpiece will be set as the origin of the workpiece coordinate system, which will be the error-free position. Error is thus determined according to this point rather than to the machine reference coordinate system.

For this reason, current methods for measuring error and creating models are limited by the following three practical issues:

(1) Kinematic parameters in the model cannot be accurately determined.

(2) Avoiding Abbe's error during the measurement of geometric error creates practical operational difficulties with conventional measurement devices.

(3) Error models including kinematic parameters contribute rotational error to overall error: inaccuracy in the

measurement of rotational error magnifies the uncertainty related to the accuracy of machine tools.

Therefore, it is necessary to establish more practical, convenient, and accurate methods of measurement and new error models for geometric error related to three-axis machine tools.

II. THE PRINCIPLE OF LOCATION-DEPENDENT ERROR MEASUREMENT

The linear axis of three-axis machine tools was structured according to kinematic stacking with each axis of motion provided a home position. For this reason, kinematic parameters between linear axis coordinate systems are necessary to describe their movement relative to one another. However, in practice, the location of the ideal motion axis line for a linear motion slide is difficult to clearly define. For this reason, it is necessary to establish a new measurement method and a geometric error model based on measurement location.

Ideally, coordinate systems for geometric error models should be established along the axis line related to the ideal motion of the linear slide to describe the spatial error caused by Abbe's error. For example, measurement of the Y linear slide, displayed in Fig. 1, has three translational errors (*EXY*, *EYY* and *EZY*) and three rotational errors (*EAY*, *EBY* and *ECY*). If, when measuring geometric error for directions x,y,z between measurement axis line (D) and ideal motion axis line (I) each have offset L_x, L_y, L_z, then the 6D component errors (EXY_d , EYY_d , EZY_d , EAY_d , EBY_d , ECY_d) for this measurement method and the results of the measurement are:

$$EXY_d = EXY + L_{x*}(1-\cos(EBY)) + L_{x*}(1-\cos(ECY)) + L_{y*}\sin(ECY) + L_{z*}\sin(EBY) \tag{1}$$

$$EYY_d = EYY + L_{x*}\sin(ECY) + L_{y*}(1-\cos(EAY)) + L_{y*}(1-\cos(ECY)) + L_{z*}\sin(EAY) \tag{2}$$

$$EZY_d = EZY + L_{x*}\sin(EBY) + L_{y*}\sin(EAY) + L_{z*}(1-\cos(EAY)) + L_{z*}(1-\cos(EBY)) \tag{3}$$

$$EAY_d = EAY \tag{4}$$

$$EBY_d = EBY \tag{5}$$

$$ECY_d = ECY \tag{6}$$

As indicated in the above explanation, when measuring rotational error (*EAY*, *EBY* and *ECY*), the measurement line is independent of the location of the measurement device; therefore, it is not necessary for the measurement device to be located on the ideal motion line *I*. However, when measuring translational error (*EXY*, *EYY* and *EZY*), the location of measurement matters; therefore, the measurement device must be placed on the ideal motion line *I*. If it is placed on line *D* in Fig. 1, then the spatial error created by rotational error will be included in the translational error. This method of measurement includes rotational error in addition to its own translational error. Therefore, the overall error ($\Delta X_d, \Delta Y_d, \Delta Z_d$) along the measurement line *D* with small-angle approximation assumptions can be expressed as follows:

$$\Delta X_d = EXY_d = EXY + L_{y*}ECY + L_{z*}EBY \tag{7}$$

$$\Delta Y_d = EYY_d = EYY + L_{x*}ECY + L_{z*}EAY \tag{8}$$

$$\Delta Z_d = EZY_d = EZY + L_{x*}EBY + L_{y*}EAY \tag{9}$$

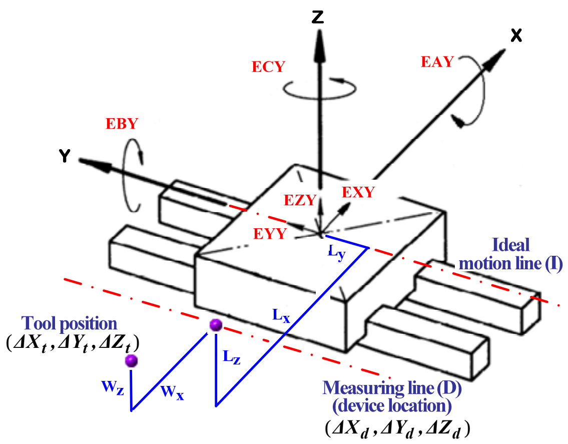

Fig. 1. Location-dependent error measurement along linear axis

Additionally, when constructing this measurement of geometric error, the kinematic parameters for L_x, L_y, and L_z have a constant value. When the linear motion axis is moved to position Y_m , the spatial error created by the rotational error at that position (*EAY*, *EBY*, and *ECY*) will each be added to the translational error (*EXY*, *EYY*, and *EZY*) and the measurement line for this measurement device can be considered the ideal motion line for the linear motion axis, meaning that rotational error includes no spatial error for any position along this measurement line. Because the error gain of rotational errors is 0, the location of measurement is the initial location of rotational error. Furthermore, in actual cutting and measuring, a location on the workpiece will be specified as the origin of the workpiece coordinate system. Set up as an error-free location, all location error on the workpiece is no longer considered error with respect to the geometric error model constructed for the ideal motion line of the machine, rather it is considered error with respect to this point. For this reason, this measurement method has practical application value.

The figure also shows the change in the actual position of the tool as it moves along the other two machine motion axes under three-axis machine tools, such that the position of the tool is no longer where it was when it was measured by the measurement device. At this point, the X and Z positions of three-axis machine tools have reached positions W_x and W_z . That is, as X and Z move to the actual cutting positions X_m and Z_m on the workpiece, and the overall error $(\Delta X_t, \Delta Y_t, \Delta Z_t)$ at the tool tip, according to the measurement devices designed to measure geometric errors using measurement axis line (D), can be expressed as follows:

$$\Delta X_t(X_m, Z_m) = EXY_d + W_{z*} EBY_d \quad (10)$$

$$\Delta Y_t(X_m, Z_m) = EYY_d + W_{x*} ECY_d + W_{z*} EAY_d \quad (11)$$

$$\Delta Z_t(X_m, Z_m) = EZY_d + W_{x*} EBY_d \quad (12)$$

where X_m and Z_m are the servo-controlled position of the X and Z servo-axis, respectively.

III. Location-Dependent Measurement of Translational and Rotational Error

Existing measurement tools are already capable of measuring six types of geometric component error (three translational and three rotational) [6] in a linear motion slide. The main purpose of this section is to apply the principle of measurement described earlier to a few commonly used measurement devices, such as straight gauges, gauge blocks, and indicators (or probes), for application in a three-axis machine tool. Combined with methods for measuring location related error, the six component errors were measured along the linear axes of a three-axis machine tool. The location, as measured in a perpendicular direction (X-axis of Fig. 1), along such a linear axis (Y-axis in Fig. 1) can be regarded as an error-free reference location. In the following, measurement of the linear Y-axis is used to illustrate the method of measuring geometric error along linear axes.

A. *Measurement of Positioning Error (EYY_d)*

The measurement devices are set up as in Fig. 2, according to (8) when positioning error is measured along the linear Y-axis slide, following the corresponding steps of measurement described below:

Step 1: Install a high-precision indicator or probe along the axis of linear motion.

Step 2: Place a high-precision gauge block on the machine platform.

Step 3: Use the high-precision indicator or probe to ensure that the linear movement remains parallel to the reference plane of the gauge block.

Step 4: Position the first measuring plane of the gauge block close to the home position of the linear motion axis, and set the positioning error (EYY_d) along the linear axes at this position to 0.

Step 5: Use an NC program to automatically measure error at each position of the gauge block, such that the indicator or probe automatically records the measurement and compares the results to the readings on the optical scale from the linear motion axis to check for geometric errors.

Step 6: The results measuring geometric error at these positions are set as positioning error (EYY_d) along the linear axis in this setup (status of the gauge block when the three-axis machine tools is at such an X-axis location).

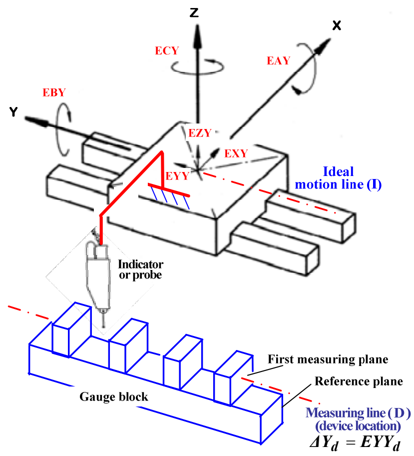

Fig. 2. Positioning error measurement

B. *Measurement of Horizontal Straightness Error (EXY_d)*

The measurement devices are set up as in Fig. 3, according to (7) when positioning error is measured along the linear Y-axis slide, following the corresponding steps of measurement described below:

Step 1: Install a high-precision indicator or probe along the axis of linear motion.

Step 2: Place a high-precision straight gauge on the machine platform.

Step 3: Use the high-precision indicator or probe to ensure that the reference surface of the straight gauge remains parallel to the ideal correction of the axis of movement.

Step 4: Return the linear motion axis to the home position, and set the horizontal straightness error (EXY_d) along the linear axes at this position to 0.

Step 5: Use an NC program to automatically measure horizontal straightness error at each position along the linear motion axis, such that the indicator or probe automatically records the measurements.

Step 6: The measurement results are horizontal straightness error (EXY_d) along the linear axis in this setup.

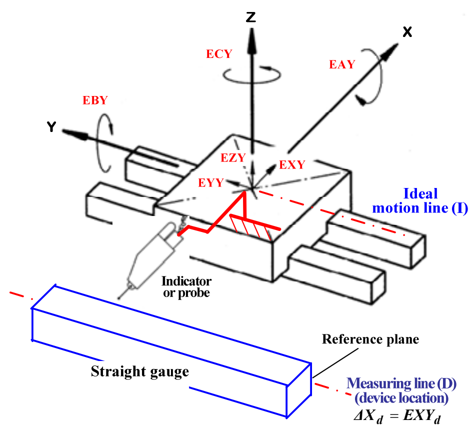

Fig. 3. Measurement of horizontal straightness error

C. *Measurement of vertical straightness error (EZY_d)*

The measurement devices are set up as in Fig. 4, according to (9) when positioning error is measured along the linear Y-axis slide, following the corresponding steps of measurement described below:

Step 1: Install a high-precision indicator or probe along the axis of linear motion.
Step 2: Place a high-precision straight gauge on the machine platform.
Step 3: Use the high-precision indicator or probe to ensure that the reference surface of the straight gauge remains parallel to the ideal correction of the movement axis.
Step 4: Return the linear motion axis to the home position, and set the horizontal straightness error (EZY_d) along the linear axes at this position to 0.
Step 5: Use an NC program to automatically measure vertical straightness error at each position of linear motion, such that the indicator or probe automatically records the measurements.
Step 6: The measurement results are the vertical straightness error (EZY_d) along the linear axis in this setup.

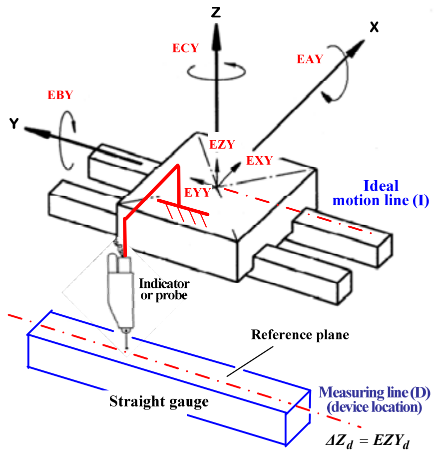

Fig. 4. Measurement of vertical straightness error

D. *Measurement of pitch error (EAY)*

The principle underlying the measurement of pitch error is based on the application of the measurement method and data from the aforementioned positioning error (EYY_d), to which we add an extension bar (L) in the vertical direction before it is used to measure the error in positioning, as shown in Fig. 5. The measurement result ΔY_S can be expressed using the following equation:

$$\Delta Y_S = EYY_d + L{*}EAY \tag{13}$$

such that, under the condition of this measurement, the amount of pitch error EAY is

$$EAY = (\Delta Y_S - EYY_d)/L \tag{14}$$

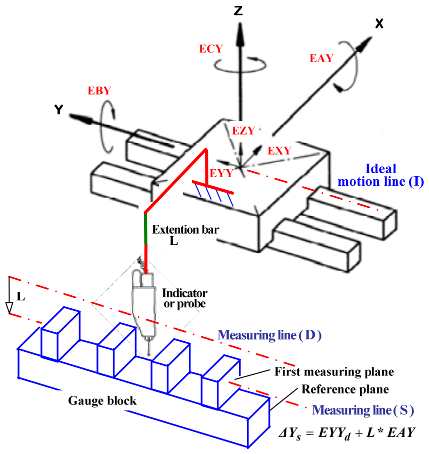

Fig. 5. Measurement of pitch error

E. *Measurement of roll error (EBY)*

The measurement of roll error is based on the application of the measurement method and data from the aforementioned vertical straightness error (EZY_d), to which we add an extension bar (L) in the horizontal direction, as shown in Fig. 6, before it is used to measure the error in vertical straightness. The measurement result ΔZ_S can be expressed using the following equation:

$$\Delta Z_S = EZY_d + L{*}EBY \tag{15}$$

such that, under the condition of this measurement, the amount of roll error EBY is

$$EBY = (\Delta Z_S - EZY_d)/L \tag{16}$$

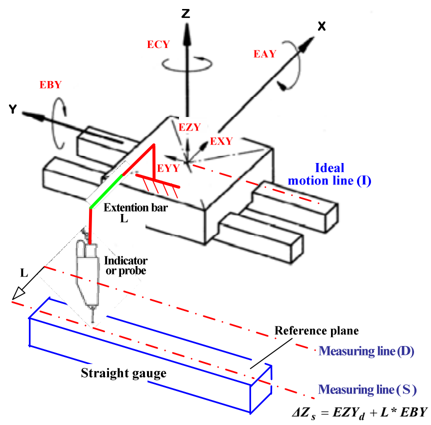

Fig. 6. Measurement of roll error

F. Measurement of yaw error (ECY)

The principle of measuring yaw error is based on the application of the measurement method and data from the aforementioned error in horizontal straightness (EXY_d), to which we add an extension bar (L) in the horizontal direction, as shown in Fig. 7, before it is used to measure error in horizontal straightness. The measurement result ΔX_d can be expressed using the following equation:

$$\Delta X_d = EXY_d + L*ECY \tag{17}$$

such that, under the condition of this measurement, the amount of yaw error ECY is

$$ECY = (\Delta X_d - EXY_d) / L \tag{18}$$

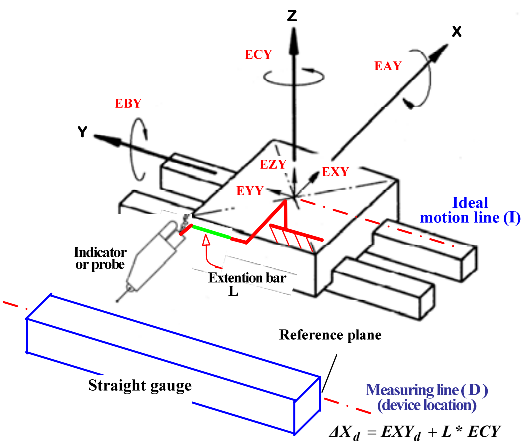

Fig. 7. Measurement of yaw error

Because the location (perpendicularity) error between the two linear axes is rotational error, the measurement method is independent of the location of the measurement device, referring to the ISO230 standard directly [13].

IV. KINEMATIC PARAMETER-INDEPENDENT ERROR MODELS

A. Defining Geometric Error for linear Axes

Definitions in ISO230 relate to standards for error inspection related to CNC machine tools, including the definition of geometric error and methods for testing. A single linear motion axis is defined as possessing six types of component error (three translational and three rotational), and location (perpendicularity) error between two axes of linear motion. According to the above definitions, a three-axis machine tool would be susceptible to 21 geometric errors.

To describe the overall geometric error of three-axis machine tools, it is necessary to establish a geometric error model for the target machine. Assuming that the structure of the machine tool is a rigid body, then a 4x4 HTM could be used to describe the relationship between each kinematic and servo control axis. The error model could scroll through the HTM of individual kinematic and driver components to determine the order of products, according to the kinematic chain of the machine [3].

Fig. 8 displays a case study of the X-axis linear motion slide. The geometric error model for kinematic parameters, location error, and component error in an X-axis linear slide is shown in the formula below, illustrating the relationship of the x coordinate system with respect to the reference coordinate system ${}^{r}T_{x}$.

(1)

$$
{}^{r}T_{x} = \begin{bmatrix} 1 & 0 & 0 & X_x \\ 0 & 1 & 0 & Y_x \\ 0 & 0 & 1 & Z_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -COX & 0 & 0 \\ COX & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -ECX & EBX & X_m + EXX \\ ECX & 1 & -EAX & EYX \\ -EBX & EAX & 1 & EZX \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}
$$

where X_x, Y_x, Z_x are the constant offset positioned at the x home with respect to the reference coordinate system in the x,y,z direction, respectively, or the kinematic parameter for the X-axes linear slide. COX is the location error between the linear X axis and an ideal linear axis (in this example, Y-axis of the reference coordinate system) which causes a small angular rotation between the two coordinate systems in the Z axial direction. EXX, EYX, EZX, EAX, EBX and ECX are the six component errors for the linear X axis, and X_m is the servo-controlled position of the X-axis slide.

The order of products for the kinematic parameter matrix, the location (perpendicularity) error matrix, and the 6D component error matrix in the above formula depend upon the pattern arrangement in the kinematic chain of the linear X axis. First, the third HTM represents the 6D component error matrix for the X axis linear slide. Second, assuming that when the X-axis slide moves to the home position the Z-axis of the X coordinate system is identical to the Z-axis of the reference coordinate system, then perpendicular error COX exists between the ideal motion axis (the X-axis of the X coordinate system) and the Y-axis of the reference coordinate system, as does the perpendicular error matrix. Finally, when the slide on the X axis moves to the X home position, the X axis slide having the kinematic parameter matrix for the origin coordinate offsets.

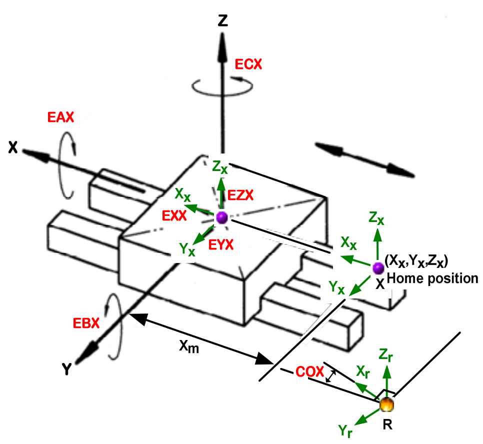

Fig. 8. Definition of linear geometric error along the X axis

B. Geometric Error Modeling

For an ideal three-axis machine tool, each position of the tool (X_w, Y_w, Z_w) and orientation of the tool (I_w, J_w, K_w) on the workpiece coordinate system has a corresponding drive position to cut the required work pieces, and tool orientation can only be defined in the (0,0,1) direction. Figure 9 presents a common three-axis machine tool (Coordinate Measuring Machine, CMM) and the definition of its coordinating system. The kinematic chain of the machine is linked by several components and three linear motion axes. One end of the chain is a tool holder. The spindle block is mounted on the Z-slide, which moves vertically using a prismatic joint. The Z-slide is bolted to the X-slide, which is stacked on the Y-slide, making the three linear axes (x,y,z) perpendicular to each other. The Y-slide is then moved on the bed with a prismatic joint. Finally, according to the definition of ISO230 and the kinematic chain sequence of this machine, the location (perpendicularity) errors are *COX*, *BOZ*, and *AOZ*.

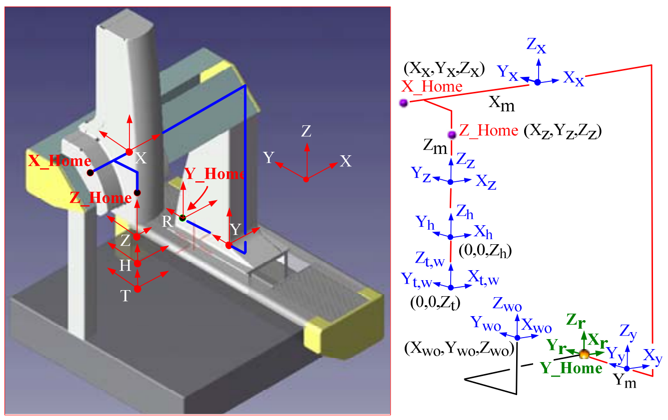

Fig. 9. Three-axis machine tools

Based on Fig. 9, the spatial relationship between the tool coordinate system and the reference coordinate system can be obtained using the formula below.

$$ {}^{r}T_t = {}^{r}T_y \; {}^{y}T_x \; {}^{x}T_z \; {}^{z}T_h \; {}^{h}T_t \tag{20} $$

The spatial relationship between the workpiece coordinate system and the reference coordinate system can be obtained using the formula below.

$$ {}^{r}T_w = {}^{r}T_{wo} \; {}^{wo}T_w \tag{21} $$

Figure 10 illustrates that, with an ideal machine, the tool coordinate system should provide identical points as those provided by workpiece coordinate system. However, actual machines cause geometric errors, so the position of the origin of the tool coordinate system with respect to the reference coordinate system $\mathbf{P_t} = [X_t \;\; Y_t \;\; Z_t]$ is obtained using the formula below.

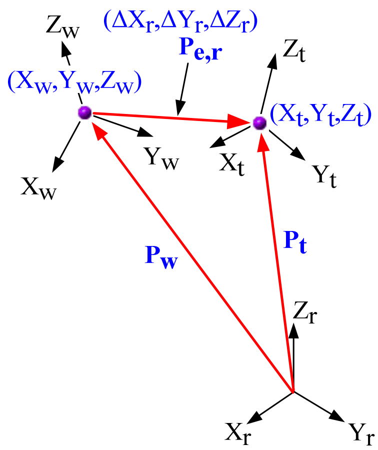

Fig. 10. Overall error at the tool end

$$ [\mathbf{P_t} \;\; 1]^T = {}^{r}T_t [0 \;\; 0 \;\; 0 \;\; 1]^T \tag{22} $$

The origin position of the workpiece coordinate system with respect to the reference coordinate system $\mathbf{P_w} = [X_w \;\; Y_w \;\; Z_w]$, can be obtained using the formula below.

$$ [\mathbf{P_w} \;\; 1]^T = {}^{r}T_w [0 \;\; 0 \;\; 0 \;\; 1]^T \tag{23} $$

Now, the position error for the tool coordinate system with respect to the workpiece coordinate system in the reference coordinate system $\mathbf{P_{e,r}}(\Delta X_r, \Delta Y_r, \Delta Z_r)$ can be obtained using the formula below.

$$ \mathbf{P_{e,r}} = \mathbf{P_t} - \mathbf{P_w} \tag{24} $$

Orientation error in the reference coordinate system $\mathbf{O_{e,r}}(\Delta I_r, \Delta J_r, \Delta K_r)$ can be obtained using the three formulas listed below.

$$[\mathbf{O_w} \quad 0]^T = ({}^rT_w - {}^rT_{w,ideal}) \; [0 \;\; 0 \;\; 1 \;\; 0]^T \tag{25}$$

$$[\mathbf{O_t} \quad 0]^T = ({}^rT_t - {}^rT_{t,ideal}) \; [0 \;\; 0 \;\; 1 \;\; 0]^T \tag{26}$$

$$\mathbf{O_{e,r}} = \mathbf{O_t} - \mathbf{O_w} \tag{27}$$

where ${}^rT_{w,ideal}$ and ${}^rT_{t,ideal}$ are the HTM for the workpiece coordinate system and tool coordinate system with respect to the reference coordinate system for ${}^rT_w$ and ${}^rT_t$, respectively, when geometric errors are not considered (the ideal machine).

Using the assumption of small-angle approximation and that second-order errors are negligible, a consolidation of geometric errors for the error model used with this three-axis machine tool is displayed in Table 1. The overall error in the direction of X, ΔX_r, is the product of each error multiplied by the error gain of each error. For example, the error contribution in the direction of X in *ECX* is –*ECX**Y_Z. This table, which is considered a geometric error sensitivity analysis table, indicates that translational errors (*EXX*, *EYX*, *EZX*, *EXY*, *EYY*, *EZY*, *EXZ*, *EYZ*, and *EZZ*) are machine kinematic parameter-independent, while rotational errors (*EAX*, *EBX*, *ECX*, *EAY*, *EBY*, *ECY*, *EAZ*, *EBZ*, *ECZ*, *COX*, *AOZ*, and *BOZ*) are machine kinematic parameter-dependent.

C. Error Model and Compensation Model Using the Combined Measurement Method

To apply the measurement devices and principles described above to three-axis machine tools, we installed an indicator (or probe) on the tool holder on the spindle of the machine in Fig. 9 to provide individual measurements of the six component errors involved in linear motion axis and the location (perpendicular) error [13] for the three linear axes. For example, when the 6D component errors were measured for linear motion along the Y axis, we first located the gauge block and straight gauge in the middle position of the X stroke, which were set as the zero error position, and then installed an indicator (probe) on the tool holder on the spindle of the machine to perform the measurements. At this point, because the measurement position of the device would be susceptible to Abbe's error, the 6D measurement results along the Y axis included all errors created by the kinematic parameters of the machine. Next, we measured the component errors for the other two linear motion axes according to the principles described above.

Applying the new measurement method to the three-axis CNC machine tool enabled us to simplify the original geometric error model containing kinematic parameters shown in Table 1 to the kinematic parameter-independent Table 2. For instance, when measuring the six component errors in linear motion along the X axis, there were three contributors to overall error ΔZ_r (*EZX*, *EAX* and *EBX*), the contributing factors of which were 1, $Y_Z, -X_Z$. Under the premise that the machine possesses positioning repeatability, we can assume that when the slide on the X axis is located at a specified position, the $Y_Z, -X_Z$ kinematic parameter will be a constant. Due to the fact that the indicator (probe) was installed at the tool end of the spindle, the error contribution of *EAX**Y_Z and *EBX**$(-X_Z)$ is reflected in *EZX*. For this reason, these two kinematic parameters can be set to zero, and their other errors can be simplified in this manner.

As illustrated in Table 2, all nine translational errors (*EXX*, *EYX*, *EZX*, *EXY*, *EYY*, *EZY*, *EXZ*, *EYZ* and *EZZ)* contributed to overall error at the tool end, but only five of the rotational errors (*EAX*, *EBX*, *EAY*, *EBY* and *ECY*) contributed, while four (*ECX*, *EAZ*, *EBZ* and *ECZ*) did not. Therefore, only 17 (21-4) geometric errors needed to be measured in this model, as shown in Table 2.

Constructing a kinematic parameter-independent three-axis geometric error model and measurement method based on the above measuring method is both practical and accurate. Furthermore, compensating for persistent geometric errors can also be accomplished using this geometric error model to establish a compensation model for three-axis geometric errors. When the three-axis machine tool is moved to ***u****(x,y,z)* positions and tool end spatial errors are ***du***, the compensation applied by the kinematic parameter-independent error compensation model is –***du***. Finally, the machine axis errors for x,y, and z motion can be corrected through a controller, and returned to their ideal position at $\mathbf{u_c}$.

TABLE 1. Error model and sensitivity analysis

	$\triangle X_r$	$\triangle Y_r$	$\triangle Z_r$	$\triangle I_r$	$\triangle J_r$	$\triangle K_r$
Error	Error gain					
EXX	1	0	0	0	0	0
EYX	0	1	0	0	0	0
EZX	0	0	1	0	0	0
EAX	0	-Zh-Zt-Zm-Zz	+Yz	0	-1	0
EBX	+Zh+Zt+Zm+Zz	0	-Xz	1	0	0
ECX	-Yz	+Xz	0	0	0	0
EXY	1	0	0	0	0	0
EYY	0	1	0	0	0	0
EZY	0	0	1	0	0	0
EAY	0	-Zh-Zt-Zm-Zz-Zx	+Yz+Yx	0	-1	0
EBY	+Zh+Zt+Zm+Zz+Zx	0	-Xz-Xm-Xx	1	0	0
ECY	-Yz	+Xz+Xm+Xx	0	0	0	0
EXZ	1	0	0	0	0	0
EYZ	0	1	0	0	0	0
EZZ	0	0	1	0	0	0
EAZ	0	-Zh-Zt	0	0	-1	0
EBZ	+Zh+Zt	0	0	1	0	0
ECZ	0	0	0	0	0	0
COX	-Yz	+Xz+Xm	0	0	0	0
AOZ	0	-Zh-Zt-Zm	0	0	-1	0
BOZ	+Zh+Zt+Zm	0	0	1	0	0

TABLE 2. Error model with kinematic parameter-independence

	$\triangle X_r$	$\triangle Y_r$	$\triangle Z_r$	$\triangle I_r$	$\triangle J_r$	$\triangle K_r$
Error	Error Gain					
EXX	1	0	0	0	0	0
EYX	0	1	0	0	0	0
EZX	0	0	1	0	0	0
EAX	0	-Zm	0	0	-1	0
EBX	Zm	0	0	1	0	0
ECX	0	0	0	0	0	0
EXY	1	0	0	0	0	0
EYY	0	1	0	0	0	0
EZY	0	0	1	0	0	0
EAY	0	-Zm	0	0	-1	0
EBY	Zm	0	-Xm	1	0	0
ECY	0	Xm	0	0	0	0
EXZ	1	0	0	0	0	0
EYZ	0	1	0	0	0	0
EZZ	0	0	1	0	0	0
EAZ	0	0	0	0	-1	0
EBZ	0	0	0	1	0	0
ECZ	0	0	0	0	0	0
COX	0	Xm	0	0	0	0
AOZ	0	-Zm	0	0	-1	0
BOZ	Zm	0	0	1	0	0

V. Conclusion

Three-axis geometric error models derived using traditional methods all set the machine reference coordinate systems at a fixed point on the base of the machine and depend on the machine kinematic chain to derive a kinematic parameter-dependent model. For practical applications, this dependence makes accurate kinematic parameters impossible to obtain, the operation of measurement devices is inconvenient, and overall error is overvalued. For this reason, this paper created a measurement method integrating "modeling, measurement, and compensation for geometric error model of three-axis machine tools using a kinematic parameter-independent" technique. This technique, integrating simple measurement methods, was used to construct a corresponding three-axis geometric error model and compensation model. The geometric error model is machine kinematic parameter-independent, making it a practical, convenient, and accurate method of measurement.

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