

A Modified Preventive Maintenance Model with Failure Rate Reduction under Warranty Consideration

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Abstract—For deteriorating and repairable equipments, there exist some different preventive maintenance (PM) models over a finite time span with failure rate reduction. Among these failure-rate-reduction PM models, it is found that the modified failure-rate-reduction PM model has the lowest expected total maintenance cost, i.e., it has the best optimal PM policy. Moreover, in the real world, equipment vendors usually provide a warranty policy for new equipments. Therefore, it is worthwhile to study how the expected total maintenance cost changes when applying different warranty policies into the modified failure-rate-reduction PM model (called the PM-warranty models). The purpose of this research is to explore the optimal PM-warranty policy for the modified failure-rate-reduction PM model with different warranty consideration. The PM cost is assumed to be variable, which is affected by the age and the amount of failure rate reduction. Three cases of PM-warranty policies are studied and compared in the users' point of view. The first case is no warranty provided during the whole PM cycles; the second case is no PM in the warranty period and the third case is having PM in the warranty period. Both the second and the third cases have PM beyond the warranty period. In this paper, the algorithm for searching the optimal solution for each case is presented. Examples with Weibull failure distribution are given to compare the optimal solution of the three cases. It is found that the third case (having PM in the warranty period) has the best optimal PM policy.

Keywords—preventive maintenance model; failure rate reduction; warranty; finite time period.

I. INTRODUCTION

The preventive maintenance (PM) can slow down the aging process and restore the system to a younger state for a deteriorating and repairable system (Pham and Wang [1], Nakagawa [2]). In real world, a system's useful life is normally finite and the replacing new system may not have exactly the same conditions (such as characteristics, investment cost, and maintenance expenses) as the old system [3] while only a few PM models are defined in a finite time period in the literature. The PM models of the leasing equipments are the most seen examples for the finite-time-period case. Pongpech and Murthy [4] developed a periodic PM model in a finite time period with failure rate reduction for the leasing equipments. Cheng and Liu [5] discussed the cases

of fully-periodic and partially-periodic PM policies using the algorithm proposed by Pongpech and Murthy [4].

For a failure-rate-reduction PM model, it can be found that a shorter time interval of PM (T) can result in a better expected total maintenance cost (TC) [6]. In literature, the "original" optimal policy of a failure-rate-reduction PM model in the finite time period (L) is obtained by searching the value of T for any given number of PM (N) over the specified range $[L/N, L/(N+1))$ which has the minimal TC as illustrated in Fig. 1. However, the "original" model does limit the possibility of finding a smaller (better) TC than its optimal solution since the value of T is limited in the range of $[L/N, L/(N+1))$ (Pongpech and Murthy [4]). Cheng et al. [6] presented a "modified" PM model with failure rate reduction by releasing the constraint of the searching range of T as illustrated in Fig. 2. Cheng et al. [6] found that the optimal PM policy of the "modified" PM model is better than the "original" PM model.

In real situation, equipment vendors usually provide warranty service for a new equipment. It is worthwhile to investigate the PM model with different warranty consideration. Therefore, the purpose of this research is to develop the optimal PM-warranty policy for the modified failure-rate-reduction PM model with different warranty consideration in a finite time span by minimizing the expected total maintenance cost. In this paper, three cases of PM-warranty models are studied in the equipment owners' point of view. The first case is no warranty provided during the whole PM cycles; the second case is no PM in the warranty period

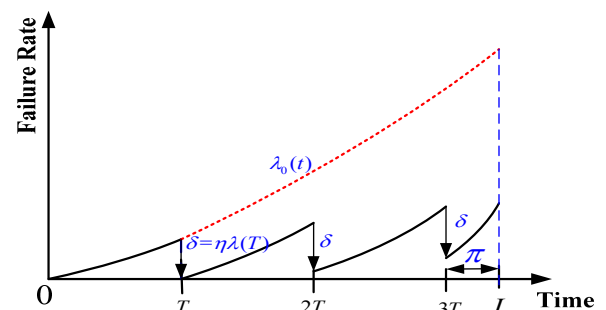


Fig. 1. The illustration of the original PM Model with $N=3$.

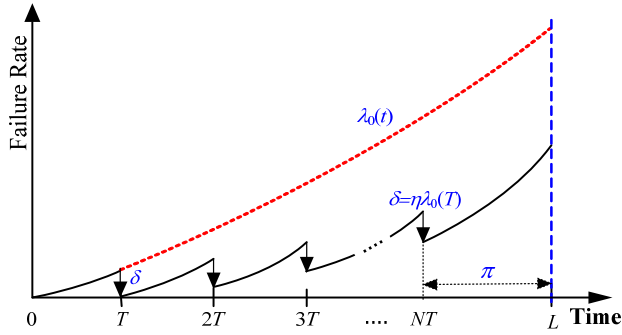


Fig. 2. The illustration of the modified PM Model (same as Case 1).

and having the PM after warranty expired; the third case is having PM in the warranty period and the period after the warranty expired.

The decision variables of the modified PM-warranty model include the PM interval (T), the number of PM (N) and the restoration factor (η). In this paper, we first develop the proposed three PM-warranty models. Next, we present the algorithm for searching the optimal solution which can include both fully-periodic and partially-periodic PM policies. Then, the optimal solutions of the examples with Weibull failure distribution are illustrated and compared for the three PM-warranty models. Finally, the sensitivity of the parameters to the decision variables and the expected total maintenance cost are analyzed.

II. THE MODIFIED PM-WARRANTY MODELS

The modified PM-warranty model for the first case (i.e., no warranty provided during the whole PM cycles) is presented as a base model and the modified PM-warranty model for the second and the third cases (i.e., having warranty) are then developed and compared with the base model accordingly.

A. Nomenclature

L	The useful life time (finite time period) for the system or equipment.
T	The time interval of each PM.
N	The number of PM performed in the finite life time period (L).
$\lambda(t)$	Failure rate function before the 1 st PM.
$\lambda_i(t)$	Failure rate function at time t where t is in the i^{th} PM cycle and $\lambda_0(t)=\lambda(t)$
$A(t)$	The expected number of failures in time t without warranty or after warranty period.
π	The time interval between the N^{th} PM and L , i.e., $\pi = L - NT$.
w	The length of warranty period, where $w < L$.
n_w	The number of PM performed within the warranty period.

η The restoration factor for measuring the amount of restoration after each PM.

$\delta_N(T, \eta)$ The amount of the failure rate reduced after each PM when given N , which is constant in each PM and is determined by the restoration factor η and $\lambda(T)$; the notation is simplified as $\delta_N(T)$ when $\eta = 1$.

C_{mr} The minimal repair cost of each random failure.

$C_{pm}(i, \delta_N(T, \eta))$ The PM cost in the i^{th} PM cycle, which is function of i and $\delta_N(T, \eta)$.

$TC(N, T, \eta)$ The expected total maintenance cost over the finite life time interval L , the notation is simplified as $TC(N, T)$ when $\eta = 1$.

B. The Assumptions

The following are the assumptions for the modified PM-warranty model.

- The equipment is deteriorating over time with power law increasing failure rate (IFR) in which Weibull failure distribution is assumed in this paper, i.e.,

$$\lambda(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} \quad (1)$$

where θ is the scale parameter and β is the shape parameter with $\beta \geq 1$.

- The system is disposed at specified finite time (L) without replacing a new one where the disposed system is assumed to have no salvage market value.
- The PM interval (T) is limited in the range of $(0, L]$.
- The PM can reduce the equipment's failure rate to a younger level. When given N , the amount of the failure rate reduced (δ) after each PM is assumed to be constant and is measured by the restoration factor (η) and the maximum possibly reduced amount of failure rate ($\lambda(T)$) as shown in the following equation.

$$\delta_N(T, \eta) = \eta \lambda(T) \quad (2)$$

- The minimal repair cost (C_{mr}) of each random failure is assumed to be constant.
- The cost of each PM ($C_{pm}(\cdot)$) is assumed to be variable and is defined in the following equation, which is affected by equipment age (expressed by the number of PM performed) and the amount of failure rate reduced after the PM.

$$C_{pm}(i, \delta_N(T, \eta)) = \sum_{i=1}^N a + bi + c \delta_N(T, \eta), \quad (3)$$

where a , b , and c are the coefficients of the PM cost function which, for each PM, may imply the fixed cost, the incremental cost, and the variable unit cost of restoration effect, respectively.

- The times to perform PM and minimal repair are negligible.
- The minimal repair cost is charged to the vendor if a random failure is occurred within the warranty period.
- The PM cost is charged to the equipment owner (user) if the PM is performed in the warranty period.
- The expected total maintenance cost (TC) consists of the total minimal repair cost and the total PM cost.
- The minimal repair cost (C_{mr}) and the PM cost ($C_{pm}(\cdot)$) are assumed to include the following items: labor of repair and maintenance, material and parts, and loss of downtime in production and it is assumed that $C_{pm}(\cdot) \geq C_{mr}$ in this paper.

C. The Failure Rate Function of a PM Model with Failure Rate Reduction

For a PM model with failure rate reduction, the failure rate function (also called the hazard rate function) of the i^{th} PM cycle (as illustrated in Fig. 2) is defined as

$$\lambda_i(t) = \lambda(t) - i\delta_N(T, \eta), \text{ for } i = 1, 2, \dots, N, \quad (4)$$

where $iT \leq t \leq (i+1)T$, $0 \leq \delta \leq \lambda(T)$.

D. Case 1: No Warranty in the Life Time with PM

In the modified PM-warranty model, the PM interval (T), the number of PM (N) and the restoration factor (η) are the decision variables. Case 1 is based on the modified PM model developed by Cheng et al. [6] (as illustrated in Fig. 2). The expected number of random failures, $\Lambda_1(L)$, in the finite time period is shown in (5).

$$\Lambda_1(L) = \sum_{i=0}^{N-1} \int_{iT}^{(i+1)T} \lambda_i(t) dt + \int_{NT}^L \lambda_N(t) dt. \quad (5)$$

Substituting $\lambda_i(t)$ of (5) by (4), $\Lambda_1(L)$ turns into

$$\Lambda_1(L) = \int_0^L \lambda(t) dt - N\delta_N(T, \eta) \left(L - \frac{(N+1)T}{2} \right) \quad (6)$$

Then, the expected total maintenance cost (TC) can be obtained as

$$\begin{aligned} TC_1(N, T, \eta) &= C_{mr} \Lambda_1(L) + \sum_{i=1}^N C_{pm}(i, T, \eta) \\ &= C_{mr} \left\{ \int_0^L \lambda(t) dt - N\eta\lambda(T) \left(L - \frac{(N+1)T}{2} \right) - c / C_{mr} \right\} \\ &\quad + N(a + ((N+1)b/2)) \end{aligned} \quad (7)$$

E. Case 2: No PM in the Warranty Period

Since the random failures occurred in the warranty period are repaired by the vendor, the minimal repair cost is counted after the warranty period (w). We assume that the first PM is performed at time T where $T > w$. The PM model for Case 2 is illustrated in Fig. 3. Hence, the expected number of random failures in the period of $[w, L]$, denoted as $\Lambda_2(w, L)$, is given as

$$\begin{aligned} \Lambda_2(w, L) &= \int_w^T \lambda(t) dt + \sum_{i=1}^{N-1} \int_{iT}^{(i+1)T} \lambda_i(t) dt + \int_{NT}^L \lambda_N(t) dt \\ &= \int_w^L \lambda(t) dt - N\delta_N(T, \eta) \left[L - \frac{(N+1)T}{2} \right]. \end{aligned} \quad (8)$$

The expected total maintenance cost function is shown as follows.

$$\begin{aligned} TC_2(N, T, \eta) &= C_{mr} \Lambda_2(w, L) + \sum_{i=1}^N C_{pm}(i, T, \eta) \\ &= C_{mr} \left\{ \int_w^L \lambda(t) dt - N\eta\lambda(T) \left(L - \frac{(N+1)T}{2} \right) - c / C_{mr} \right\} \\ &\quad + N(a + ((N+1)b/2)) \end{aligned} \quad (9)$$

It can be seen from the above equations that there is only small difference between Case 1 and Case 2.

F. Case 3: Having PM in the Warranty Period

In Case 3, it is assumed that the PM time interval is consistent in the entire finite time period (L), i.e., the PM is performed periodically every T unit of time and is not affected by the warranty period. Suppose the number of PM performed within the warranty period is n_w where $n_w = \lfloor w/T \rfloor$. It can be seen that $n_w = 0$ if $T > w$ and $n_w > 0$ if $T \leq w$. The PM-warranty model for Case 3 is illustrated in Fig. 4.

The expected number of random failures which are occurred in the time period $[w, L]$ is derived in (10).

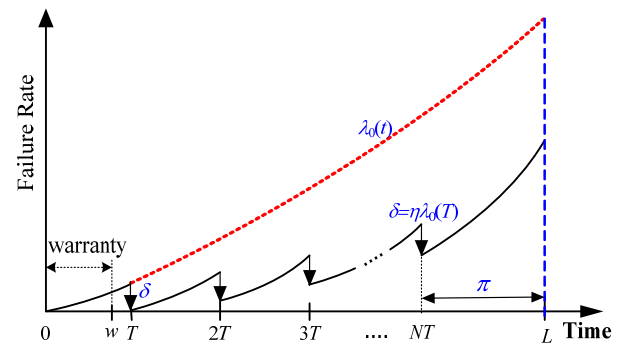


Fig. 3. The illustration of the modified PM-warranty Model for Case 2.

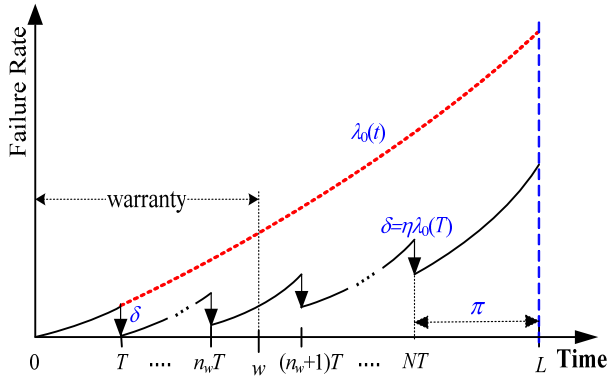


Fig. 4. The illustration of the modified PM-warranty Model for Case 3.

$$\begin{aligned} \Lambda_3(w, L) &= \int_w^{(n_w+1)T} \lambda_{n_w}(t) dt + \sum_{i=n_w+1}^{N-1} \int_{iT}^{(i+1)T} \lambda_i(t) dt + \int_{NT}^L \lambda_N(t) dt \\ &= \int_w^L \lambda(t) dt - N \delta_N(T, \eta) \left[L - \frac{(N+1)T}{2} \right] \\ &\quad - n_w \delta_N(T, \eta) \left[\frac{(n_w+1)T}{2} - w \right] \end{aligned} \quad (10)$$

The expected total maintenance cost function for Case 3 is obtained as

$$\begin{aligned} TC_3(N, T, \eta) &= \\ C_{mr} &\left\{ \int_w^L \lambda(t) dt - \eta \lambda(T) \left[NL - \frac{1}{2} [N(N+1) - n_w(n_w+1)] T - n_w w \right] \right\} \\ &+ N \left(a + \frac{(N+1)}{2} b + c \eta \lambda(T) \right) \end{aligned} \quad (11)$$

III. THE OPTIMAL PM-WARRANTY POLICIES

The optimal solution for the above PM-warranty models can be obtained by minimizing the expected total maintenance cost TC . It requires an algorithm with numerical method to search for the optimal solution. The algorithm proposed by Cheng and Liu [5] is applied in this paper since it is suitable for the modified PM models having more than one decision variable. The algorithm for Case 1 is presented below. The algorithms for Cases 2 and 3 are similar to the algorithm for Case 1 and are omitted.

A. The Algorithm for Searching the Optimal Solutions of Case 1

- 1) Let $N = 0$, $T_N = L$, $\eta_N = 0$.
- 2) Calculate $C_{min} = TC_0(N, T_N, \eta_N)$ using (7). (Note: $TC_0(\cdot)$ is the expected total maintenance cost of no PM being performed.)
- 3) Let $N = 1$.
- 4) Calculate $T_U = L/N$.

- 5) Use Nelder-Mead method to Search the values of T_N in the range of $(0, T_U]$ and η_N in the range of $[0, 1]$ such that $TC(N, T_N, \eta_N)$ shown in (7) is minimized; let $C_0 =$ minimal value of $TC(N, T_N, \eta_N)$.
- 6) If $C_0 \geq C_{min}$ then
 - obtain the optimal solution $N^* = N-1$, $T^* = T_{N^*}$, $\eta^* = \eta_{N^*}$, and $TC(N^*, T^*, \eta^*)$;
 - stop,
- else
 - let $N = N+1$ and $C_{min} = C_0$;
 - go to Step 4).

B. Numerical Examples

Numerical examples for the three cases of the modified PM-warranty PM are performed based on the following conditions: $L=5$; $w=2$; $C_{mr}=1$; and C_{pm} is defined in (3) where different values are assigned to the coefficients a , b , and c as shown in Tables I to III. The time between failure of the equipment is assumed to follow Weibull distribution with scale parameter $\theta=1$ and shape parameter $\beta=2.5$ and 3.

Table I shows the optimal solutions of the examples for Case 1 (i.e., no warranty); Table II is for Case 2 where no PM is assumed in the warranty period; Table III is for Case 3 where PM may be performed in the warranty period. It can be investigated from these tables that the 1st PM restore the failure rate to zero (i.e., $\eta^* = 1$). It means that the 1st PM of the optimal policy is a perfect maintenance.

It can be noted that the optimal solutions of Case 1 (no warranty) have larger total maintenance cost ($TC(\cdot)$) than Case 2 for all the examples. This implies that the optimal policy of no PM in the warranty period is better than the optimal policy of no warranty. The inevitable results can verify the proposed PM-warranty models and their algorithms for optimal solutions are valid.

It can be seen from Tables II and III that both Case 2 and Case 3 have the same optimal solutions when $\beta = 2.5$. This means that having PM or not in the warranty period is no difference for equipment with low risk of failure (i.e. small β). For the examples of $\beta = 3$ and the variable unit cost of restoration effect (c) is equal to or greater than 0.8, the optimal solutions for Case 3 has slightly larger $TC(\cdot)$ than Case 2. This implies that no PM in the warranty period is the best choice for equipment with high risk of failure and high cost of restoration. When analyzing the results of different values of β , we can see that examples with higher value of β usually have the optimal solutions with smaller number of PM, longer PM time interval.

In the sensitivity analysis, It is obvious that β has the highest degree of effect to the $TC(\cdot)$; parameter c has the second highest degree of effect; parameters a and b have almost no effect to the $TC(\cdot)$.

IV. CONCLUSIONS

In this paper, three modified PM-warranty models with failure rate reduction are developed. The optimal solution is obtained by minimizing the expected total maintenance cost. The algorithms for searching the optimal solution with more than one decision variable are also presented. The results of this study are helpful for making decision of equipment acquisition and maintenance policy. Thus, It is worth to further explore the theoretical properties of the proposed modified PM-warranty models.

TABLE I. THE OPTIMAL SOLUTIONS OF THE EXAMPLES FOR CASE 1.

Parameter			N		T		π		η		TC	
a	b	c	2.5	3	2.5	3	2.5	3	2.5	3	2.5	3
1	0	0	2	1	2	3.33	1	1.67	1	1	29.62	70.44
1	0.8	0	1	1	3	3.33	2	1.67	1	1	31.72	71.24
1	1.5	0	1	1	3	3.33	2	1.67	1	1	32.42	71.94
1	0	0.8	2	1	1.68	2.8	1.64	2.2	1	1	39.61	93.07
1	0.8	0.8	1	1	2.52	2.8	2.48	2.2	1	1	40.9	93.87
1	1.5	0.8	1	1	2.52	2.8	2.48	2.2	1	1	41.6	94.57
1	0	1.5	1	1	2.1	2.33	2.9	2.67	1	1	46.25	106.94
1	0.8	1.5	1	1	2.1	2.33	2.9	2.67	1	1	47.05	107.74
1	1.5	1.5	1	1	2.1	2.33	2.9	2.67	1	1	47.75	108.44
1.5	0	0	2	1	2	3.33	1	1.67	1	1	30.62	70.94
1.5	0.8	0	1	1	3	3.33	2	1.67	1	1	32.22	71.74
1.5	1.5	0	1	1	3	3.33	2	1.67	1	1	32.92	72.44
1.5	0	0.8	1	1	2.52	2.8	2.48	2.2	1	1	40.6	93.57
1.5	0.8	0.8	1	1	2.52	2.8	2.48	2.2	1	1	41.4	94.37
1.5	1.5	0.8	1	1	2.52	2.8	2.48	2.2	1	1	42.1	95.07
1.5	0	1.5	1	1	2.1	2.33	2.9	2.67	1	1	46.75	107.44
1.5	0.8	1.5	1	1	2.1	2.33	2.9	2.67	1	1	47.55	108.24
1.5	1.5	1.5	1	1	2.1	2.33	2.9	2.67	1	1	48.25	108.94

TABLE II. THE OPTIMAL SOLUTIONS OF THE EXAMPLES FOR CASE 2.

Parameter			N		T		π		η		TC	
a	b	c	2.5	3	2.5	3	2.5	3	2.5	3	2.5	3
1	0	0	1	1	3	3	2	2	1	1	25.26	64.00
1	0.8	0	1	1	3	3	2	2	1	1	26.06	64.80
1	1.5	0	1	1	3	3	2	2	1	1	26.76	65.50
1	0	0.8	1	1	2.52	2.8	2.48	2.2	1	1	34.44	85.07
1	0.8	0.8	1	1	2.52	2.8	2.48	2.2	1	1	35.24	85.87
1	1.5	0.8	1	1	2.52	2.8	2.48	2.2	1	1	35.94	86.57
1	0	1.5	1	1	2.1	2.33	2.9	2.67	1	1	40.59	98.94
1	0.8	1.5	1	1	2.1	2.33	2.9	2.67	1	1	41.39	99.74
1	1.5	1.5	1	1	2.1	2.33	2.9	2.67	1	1	42.09	100.44
1.5	0	0	1	1	3	3	2	2	1	1	25.76	64.50
1.5	0.8	0	1	1	3	3	2	2	1	1	26.56	65.30
1.5	1.5	0	1	1	3	3	2	2	1	1	27.26	66.00
1.5	0	0.8	1	1	2.52	2.8	2.48	2.2	1	1	34.94	85.57
1.5	0.8	0.8	1	1	2.52	2.8	2.48	2.2	1	1	35.74	86.37
1.5	1.5	0.8	1	1	2.52	2.8	2.48	2.2	1	1	36.44	87.07
1.5	0	1.5	1	1	2.1	2.33	2.9	2.67	1	1	41.09	99.44
1.5	0.8	1.5	1	1	2.1	2.33	2.9	2.67	1	1	41.89	100.24
1.5	1.5	1.5	1	1	2.1	2.33	2.9	2.67	1	1	42.59	100.94

TABLE III. THE OPTIMAL SOLUTIONS OF THE EXAMPLES FOR CASE 3.

Parameter			N		T		π		η		TC		n_w	
a	b	c	2.5	3	2.5	3	2.5	3	2.5	3	2.5	3	2.5	3
1	0	0	1	1	3	3	2	2	1	1	25.26	64.00	0	0
1	0.8	0	1	1	3	3	2	2	1	1	26.06	64.80	0	0
1	1.5	0	1	1	3	3	2	2	1	1	26.76	65.50	0	0
1	0	0.8	1	2	2.52	1.73	2.48	1.54	1	1	34.44	87.75	0	1
1	0.8	0.8	1	2	2.52	1.73	2.48	1.54	1	1	35.24	90.15	0	1
1	1.5	0.8	1	2	2.52	1.73	2.48	1.54	1	1	35.94	92.25	0	1
1	0	1.5	1	2	2.1	1.5	2.90	2	1	1	40.59	98.75	0	1
1	0.8	1.5	1	2	2.1	1.5	2.90	2	1	1	41.39	101.15	0	1
1	1.5	1.5	1	2	2.1	1.5	2.90	2	1	1	42.09	103.25	0	1
1.5	0	0	1	1	3	3	2.00	2	1	1	25.76	64.50	0	0
1.5	0.8	0	1	1	3	3	2	2	1	1	26.56	65.30	0	0
1.5	1.5	0	1	1	3	3	2	2	1	1	27.26	66.00	0	0
1.5	0	0.8	1	2	2.52	1.73	2.48	1.54	1	1	34.94	88.75	0	1
1.5	0.8	0.8	1	2	2.52	1.73	2.48	1.54	1	1	35.74	91.15	0	1
1.5	1.5	0.8	1	2	2.52	1.73	2.48	1.54	1	1	36.44	93.25	0	1
1.5	0	1.5	1	2	2.1	1.5	2.9	2	1	1	41.09	99.75	0	1
1.5	0.8	1.5	1	2	2.1	1.5	2.9	2	1	1	41.89	102.15	0	1
1.5	1.5	1.5	1	2	2.1	1.5	2.9	2	1	1	42.59	104.25	0	1

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REFERENCES

- [1] H. Pham, and H. Wang, "Invited review: imperfect maintenance," European Journal of Operational Research, vol. 94, pp. 425-438, 1996.
- [2] T. Nakagawa, "Periodic and sequential preventive maintenance policies," Journal of Applied Probability, vol. R-23/2, pp. 536-542, 1986.
- [3] C.Y. Cheng, X.F. Zhao, M. Chen, and T.H. Sun, "A failure-rate-reduction periodic preventive maintenance model with delayed initial time in a finite time period," Quality Technology and Quantitative Management, vol. 11, no. 3, pp. 245-254, 2014.
- [4] J. Pongpech and D.N.P. Murthy, "Optimal periodic preventive maintenance policy for leased equipment," Reliability Engineering and System Safety, vol. 91, pp. 772-777, 2006.
- [5] C.Y. Cheng and H.H. Liu, "The finite-time-period preventive maintenance policies with failure rate reduction under a warranty consideration," Journal of the Chinese Institute of Industrial Engineers, vol. 27, pp. 81-89, 2010.
- [6] C.Y. Cheng, R. Guo, M. Chen, and J.T. Chen, "A modified preventive maintenance model with failure rate reduction in a finite time span," Proceedings of the 4th Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM 2010), 2-4 December 2010, Wellington, New Zealand, pp.121-128.