The Impact of element stiffness on reliablity of reinforced concrete structures

Ivana Klaric

Faculty of Politehnic,
Zenica, BiH
ivanacurguz1@gmail.com, iva43gr@yahoo.com

Abstract—**Due to different loads, the failure of concrete structures can be different. Most failures of this construction are due to concrete failure or reinforcement failure. In this paper it is presented an example of reinforced concrete frame failure because of the different loads. Beam-sway mechanism of the construction failure is considered. Limited function is defined for ultimate limit states, for bending. Reliability indexes are different for different fracture mechanisms. Results show how reliability index is changed for different relations in rigidity of columns and beams. Reliability calculation is carried out with FORM method using "Vap" software.**

Keywords—concrete failure; reinforcement failure; failure mechanisms; reliability indexes

I. INTRODUCTION

Concrete structures are by themselves filled with defects, micro cracks and damages, therefore the failure of these constructions is sometimes unpredictable. Due to the fact that concrete resistance on pressure is much higher than on tension, tensile stress is mostly taken over the provided reinforcement. Failure of these construction can be caused by the concrete failure or reinforcement failure. Concrete is inhomogeneous material which is prone to inductile failure. Knowing this characteristic of concrete, this phenomenon should be avoided by proper arrangement of structural elements and proper reinforcement. Therefore the more ductile behavior of this structures is preferable.

During the design of reinforced concrete structures we are trying to respect the basic characteristics of stability and security and that is that the structure should be rigid, ductile enough and of adequate capacity. The capacity of the building greatly influences the formation of plastic hinges in RC structure, because with its increase, later will be created plastic joints in the most stressed areas. In order to avoid brittle failure and rapid destruction of the building it is necessary that the object is as ductile and that is able to deform in a nonlinear field without a breakdown. It is particularly important to keep in mind that due to the horizontal forces such as wind or seismic, plastic hinges are not caused in the columns of the frame.

With the analysis of reliability of reinforced concrete structures the probability of fracture and collapse of structures is determined. The most important step in determining the reliability is proper defining the limit functions and determination of all the weaknesses of the structure which could cause failure. With concrete structures that fracture usually occurs due to excess stress of bending, shear and torsion.

II. RELIABILITY OF STRUCTURES

Reliability of reinforced concrete structures depends on the used reinforcement, concrete quality and loads imposed on the building. Reliability can be quantified with the help of mathematical probability theory and statistics. If the failure of building is marked with p_f (probability of failure), then the reliability is expressed with mark r (reliability) and then follows the relation [1]:

$$r = 1 - p_f \tag{1}$$

The problem of determining the reliability of the building is in fact a problem of determining the probability of failure that occurs in those cases when the load S (stress) is greater than the resistance of construction itself R (resistance).

$$p_f = P[G(R,S) \le 0] = P[R - S \le 0] = \int_D f_{RS}(R,S)\,dRdS \tag{2}$$

Here the f_{RS} is joint density distribution for resistance and the load. In the majority of cases of the density distribution cannot be determined, nor is it possible to make the integration of this function by a domain of failure. This problem of determining the failure probability is further complicated if the limit function is dependent on the time, as is the case with loads that are dynamic in nature and which directly depend on the function of time [2].

$$p_f = \int_{0<t\le T} P\big[\min g(x, y(t)) \le 0|x\big] f(x)\,dx \tag{3}$$

A. Form Method (First Order Second Moment Reliability Method)

Determining the probability of fracture using the reliability index is enabled by FORM method which is defined that the probability of failure is equal to[1] [2]:

$$p_f = \Phi\left(\frac{Z - \mu_Z}{\sigma_Z}\right) = \Phi(-\beta) \tag{4}$$

Where Z is a limit function, μ_Z and σ_Z are the mean values and standard deviation of the limit function. Limit function is approximated by the Taylor first order [2] where G(X) is the limit function, X is the vector of random variables and $\overline{x_i}$ is the mean of the variable x_i.

$$Z = G(X) \tag{}$$

$$Z = G(\overline{x}) + \sum_{i=1}^{n} \frac{\partial G}{\partial x_i}(x_i - \overline{x_i}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 G}{\partial x_i \partial x_j}(x_i - \overline{x_i})(x_j - \overline{x_j}) + \cdots. \tag{5}$$

In the formulation (4) the β is reliability index. The reliability index multiplied by the standard deviation $\beta\sigma_Z$ represents measure of the mean value distance from the beginning of the coordinate Z =0, and in this way we obtain a point called calculation point which separates the failure domain from reliability domain. FORM method converts all the random variables that do not follow the Normal distribution into the variables that follow a Normal distribution, and only with such input data of all random variables defined in the limit state we can determinate the mean and standard deviation [2]. This method of determining the structure failure has its advantages and disadvantages. The main advantage of this method is that the probability of failure can be determined without knowing the common density distribution, and integration of that distribution by the failure domain is not needed.

The disadvantages of this method are that sometimes it is impossible to convert all of the random variables that do not follow the Normal distribution into the Normal distribution. Another disadvantage lies in the fact that sometimes the limit function cannot be approximated with Taylor's first order, i.e. it is not possible to ignore the members of a higher order because it would be unrealistic date on structure failure.

B. Hasofer Lind Form Method (First Order Second Moment Reliability Method)

Given the fact that the classic FORM method had many disadvantages Hasofer and Lind had modified this method. Hasofer-Lind method is a method which all the random variables converts into a standard Normal variables N (0,1) with a mean value equal to 0 and standard deviation equal to the 1.

$$x_i' = \frac{x_i - \overline{x_i}}{\sigma_{vi}} \tag{6}$$

Also the joint density distribution is then a function of the Normal variables with the property N (0.1) $f_x(x)$ and the limit function G (X'). If all the variable x_i are mutually independent random variables that follow a Normal distribution it is easy to

calculate the reduced value with properties of N(0.1), however, if they are mutually interdependent the problem gets complicated and comes down to a search for those values of x* which are independent and such are translated in the form x'

For linear limit function the following formulations are applied:

$$R' = \frac{R - \mu_R}{\sigma_R}; \ S' = \frac{S - \mu_S}{\sigma_S} \tag{7}$$

$$R - S = R'\sigma_R + \mu_R - S'\sigma_S - \mu_S = 0 \tag{8}$$

$$\beta_{HL} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{9}$$

Minimum distance of the limit function from the origin then represents β_{HL}.

$$p_f = \Phi(-\beta_{HL}). \tag{10}$$

In most cases, the limit function is not linear but is of a higher order function. In this case the problem is solved by finding the "design point" Fig.1 which is a point on the limit function that is at least away from the origin. If this minimum distance can be found it is easy to calculate the probability of failure that is the reliability. Reliability index is a constant value, bearing in mind that no matter how function of the limit state looks its geometric shape and distance from the origin remains constant.

The calculation of the minimum distance becomes an optimization problem.

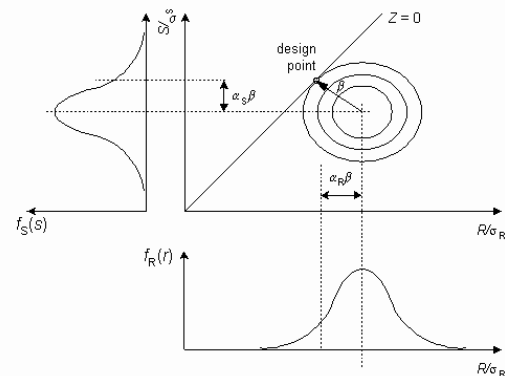$$D = \sqrt{(X')^T(X')} \tag{11}$$

$$G(X') = 0 \tag{12}$$

Fig. 1. Hasofer Linds Method of Reliability

The reliability index in this case looks like:

$$\beta_{HL} = -\frac{\sum_{i=1}^{n} x'_{di}\left(\frac{\partial G}{\partial x'_{di}}\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial G}{\partial x'_{di}}\right)^2}} \tag{13}$$

The calculation point and sensibility factors then look:

$$x'_{di} = \alpha_{di}\beta_{HL} \tag{14}$$

$$\alpha_{di} = \frac{\left(\frac{\partial G}{\partial x'_{di}}\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial G}{\partial x'_{di}}\right)^2}} \tag{15}$$

Sensibility factors α_{di} represent cosine of angles between the individual axes of a random variables with the direction of reliability index and represent the impact of certain variables on the reliability [3].

If the random variable does not follow the Normal distribution, it is necessary to convert it to a Normal distribution, and then apply

$$\mu_{xi}^{N} = x_{di} - \phi^{-1}[F_i(x_{di})]\sigma_{xi}^{N} \tag{16}$$

$$\sigma_{xi}^{N} = \frac{\bar{\phi}\{\phi^{-1}[F_i(x_{di})]\}}{f_i(x_{di})} \tag{17}$$

Where f_i and F_i are density distribution and cumulative distribution of the random variable that does not follow a Normal distribution, $\bar{\phi}$ and ϕ are the density distribution and cumulative distribution of a Normal random variable [2]. Today there are various methods of converting into the Normal distribution among which is the approximate method of Paloheimo-Hannus [3].

C. Crude Monte Carlo Method

Monte Carlo simulation is a method that is often used when an analytical solution of probability failure is not possible to determine. This method is used in some very complex problems with a large number of random variables when it is not possible to apply other methods. The essence of this method consists in counting the random variables that meet the limit function g (x) <0 with the tag N_f and with the total number of simulation which we mark with N. The downside of this method is that for sufficiently accurate result it is necessary a large number of these cycles. Then the probability of failure is equal to their relationship:

$$p_f = \frac{N_f}{N} \tag{18}$$

D. Sorm (Second Order Reliability Method)

The method of the second order is used in cases when the limit state function is not linear. There are many methods of the second order used today, one of these is the method which the curve of the limit function around the point which is minimum distance from the origin approximates with the function of the second order, i.e. quadratic function. In the Equation (19) k_i is the main curvature of the limit function in „design point".

Probability of failure by Breitung is approximately equal to:

$$p_f \approx \Phi(-\beta) \prod_{i=1}^{n-1} \frac{1}{\sqrt{(1+\beta k_i)}} \tag{19}$$

III. RELIABILITY OF REINFORCED CONCRETE STRUCTURES

A. Failure Mechanisms

The representation of determination of the reliability index and failure of structures will be considered on a reinforced concrete frame on Fig. 2.
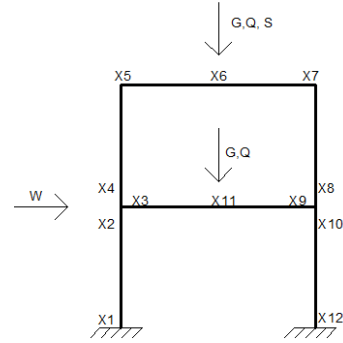
Fig. 2. Reinforced frame with loads

We will assume that the frame is loaded by its own weight, variable load, with snow and horizontal forces of wind and by the recommendations of the [4]. Since the wind force and snow force dependent on the time factor this fact will be taken into account in the design.

The frame, for simplicity of the design is taken as a simple, two-storey, six times static indefinite.

$$\sum_{i=1}^{k} P_i \, \delta\varepsilon_i = \sum_{j=1}^{m} M_j^{\bullet} \delta\varphi_j \tag{20}$$

Where $\sum_{i=1}^{k} P_i \, \delta\varepsilon_i$ is work of external forces and $\sum_{j=1}^{m} M_j^{\bullet} \delta\varphi_j$ is work on internal forces on virtual displacement.

Based on the previous formulation (20) we can write the limit function for the combined sway-beam mechanism [5].

$$G_3 = m_r \cdot (X_1 + 2X_6 + 2X_{11} + X_7 + X_9 + X_{12}) - m_e \cdot (h * W + 2 \cdot 0.5 \cdot a_1 \cdot (G + Q) + 0.5 \cdot a_1 \cdot S) \leq 0 \tag{21}$$

In the equation (21) X_i represent the places of plastic joints, and W, G, Q and S are wind, permanent load, variable load and snow load.

B. Resistance

In this case it will be considered only the impact of the reliability index on the bending, where it will be ignored influence of normal forces on the bending in beams. If the concrete tensile stress is neglected and the concrete stress in the entire compression zone is assumed constant and equal to f_c then (22) can be determinated from the horizontal equilibrium of forces [5]:

$$M_p = A_a \cdot f_y \cdot \left(d_2 - a - 0.5 \cdot \frac{A_a \cdot f_y}{0.85 \cdot b_2 \cdot f_c}\right) \qquad (22)$$

In the equation (22) b_2 is beam with and other expressions are defined in Table 3. If we assume that the failure occurred entirely as a result of the failure in the reinforcement than we get a simplified expression.

$$M_p = A_a \cdot f_y \cdot (d_2 - a) \cdot 0.9$$

$$N_p = A_a \cdot f_y + 0.85 \cdot f_c \cdot b_1 \cdot d_1 \qquad (23)$$

For moments of plasticity in the columns is valid according to second order theory [4].

$$M_p = K_2 \cdot \left(A_{ac} \cdot f_y \cdot \left(\frac{d_1 - 2 \cdot a}{2}\right) + 0.85 \cdot f_c \cdot b_1 \cdot d_1{}^2 \cdot \frac{1}{8}\right)$$

$$K_2 = \frac{(N_u - N)}{\frac{(N_u - N_{bal})}{2}} \leq 1 \ i \ N_u = 0.85 \cdot b_1 \cdot d_1 \cdot f_c + A_{ac} \cdot f_y, \ N_{bal} = \frac{0.85 \cdot b_1 \cdot d_1 \cdot f_c}{2}$$

$$(24)$$

The Equation (24) is valid for $N > 0.5 \cdot 0.85 \cdot b_1 \cdot d_1 \cdot f_c$, but the other side for $N < 0.5 \cdot 0.85 \cdot b_1 \cdot d_1 \cdot f_c$ is valid (25) where N is a normal force in column.

$$M_p = \left[A_{ac} \cdot f_y \cdot \left(\frac{d_1 - 2a}{2}\right) + d_1 \cdot N \cdot \left(1 - \frac{N}{2 \cdot 0.85 \cdot b_1 \cdot d_1 \cdot f_c}\right)\right]$$

$$N_p = A_{ac} \cdot f_y + 0.85 \cdot f_c \cdot b_1 \cdot d_1 \qquad (25)$$

Modeling of random variables is shown below on the basis of guidelines JCSS [4]. Description of all resistance variables are shown on Table 3.

C. Loads

1) Permanent load

$$G = \int_V \gamma_c \, dV \qquad (26)$$

Permanente load includes its own weight of construction and the weight of unconstructive elements, where γ_c is volume density of concrete and V is volume of concrete element [4].

2) Snow

Snow load according to Eurocode EN is

$$s = \mu_i \cdot C_e \cdot C_t \cdot s_k \qquad (27)$$

Characteristic snow load depends on the position of the building, i.e., climatic conditions and altitude of the area where the building is constructed. Since each state has its own annexes on the basis of which builds policies for loads in this paper load the values of snow were taken in accordance with legislation of Bosnia and Herzegovina.

In this paper, in addition to snow, which is described with maximal Gumbel distribution, the wind is present too. As the wind is going to be modelled for construction period of 5 years we will do a similar thing for snow, and on the basis of these data determine the reliability of the return period T =50 years. Typical snow load with a return period of T =5 years and fractile of 5% is shown in the following equation [3] [6].

$$s_k = s_{k\,sr} + \frac{1}{a}[\ln N - \ln(-\ln(p))] \qquad (28)$$

Where $s_{k\,sr}$ is characteristic value of snow load on ground for 1 year, N is return period, a is scale parameter of Gumbel distribution and p is probability fractile of 5%.

3) Variable load

The main characteristics of variable load are shown on Table 1 and Table 2 according to [1] [4].

TABLE I.   LONG TERM LOAD FOR CATEGORY OF STRUCTURE

Category	A_o (m^2) correlation area	m_q (kN/m^2) mean value for uniformly distributed long term load	σ_v (kN/m^2) Standard deviation with zero mean random variable V	σ_u (kN/m^2) Standard deviation with zero mean random field U(X,Y)	$1/\lambda$ (Years) Return period
Office	20	0.5	0.3	0.6	5

TABLE II.   SHORT TERM LOAD FOR CATEGORY OF STRUCTURES

Category	A_o (m^2) correlation area	m_p (kN/m^2) mean value for uniformly distributed short term load	σ_u (kN/m^2) Standard deviation with zero mean random field U(X,Y)	$1/\nu$ (year) Return period	D_p (days) Duration of short term load

Office	20	0.2	0.4	0.3	1-3

The standard deviation of the long and short term load is:

$$\sigma_l = \sqrt{\sigma_v^2 + \sigma_u^2 \cdot k \cdot \frac{A_0}{A}} \qquad (29)$$

$$\sigma_s = \sqrt{\sigma_u^2 \cdot k \cdot \frac{A_0}{A}} \qquad (30)$$

In the Equation (29) and (30) k is reduction factor and A is influence area.

4) Wind

Wind force equation is:

$$w = c_a \cdot c_g \cdot c_r \cdot q_{ref} = c_a \cdot c_g \cdot c_r \cdot 0.5 \cdot \rho_a \cdot v^2 \quad (31)$$

Basic distribution of these factors can follow the Log-normal or Normal distributions while the wind speed (in case of maximum speed) follows Gumbel distribution. For wind speed we will take the mean 10-minute value of v = 30m / s with a coefficient of variation of 0.1, but if we want to determine the wind speed for N years, the maximum wind speed also follows Gumbel distribution and the mean value and standard deviation of such distribution can be determined on the basis of the maximum of mean values μ_1 and standard deviations for one year σ_1 [4]

$$\mu_N = \mu_1 + 0.78 \cdot \sigma_1 \cdot \ln(N), \sigma_N = \sigma_1 \qquad (32)$$

The main characteristics of the frame and their distribution are shown in the following Table 3.

TABLE III.   BASIC VARIABLES AND THEIR DISTRIBUTIONS

Random variable	Description of variable	Distribution	Parameter(m;s)
a (m)	concrete cover	Normal	0.03;0.005
a_1 (m)	with of the frame	Deterministic	4
a_2 (m)	distance between the frames	Deterministic	4
$A_{a\,s}$ (m^2)	surface of reinforcement over the support of beams	Deterministic	0.001134
A_{af} (m^2)	surface of reinforcement in the field of beams	Deterministic	0.001134
$A_{a\,c}$ (m^2)	surface of reinforcement in column	Deterministic	0.001134
B (m)	width of the pressed concrete zone	Deterministic	1.64
b_1 (m)	column dimensions (cross section)	Normal	0.35; 0.0035
b_2 (m)	beam width	Normal	0.35; 0.0035
c_a	aerodynamic factor	Normal	1.1; 0.132
c_e	exposure coefficient	Deterministic	1
c_g	gust factor	Normal	3.15; 0.378
c_r	roughness factor	Normal	0.784; 0.117
c_t	thermal coefficient	Deterministic	1

d_1 (m)	column dimension (cross section)	Normal	0.35; 0.0035
d_2 (m)	height of beam	Normal	0.35; 0.0035
e (m)	thickness of slab	Deterministic	0.12
f_c (kN/m^2)	concrete strenght	Log-Normal	30000; 5400
f_y (kN/m^2)	yield strenght	Log-Normal	400000; 24000
h_s (m)	height of column	Deterministic	3
m_e	uncertainty of loads	Normal	1; 0.2
m_q	uncertainty of wind load	Normal	0.8; 0.16
m_r	uncertainty of resistence	Normal	1.1; 0.05
μ_i	shape coefficient	Normal	0.8; 0.12
q_l (kN/m^2)	long term loads	Gamma	0.5;0.735
q_s (kN/m^2)	short term loads	Exponencial	0.2;0.29
r_a (kg/m^3)	mass density air	Deterministic	1.25
r_c (kN/m^3)	volume density of concrete	Normal	25;1
V (m/s)	ref wind speed for T=5 years	Gumbel	34; 3.4
s_k (kN/m^2)	snow on ground for return period T=5 years	Gumbel	1.3; 0.225

IV.   RELIABILITY ANALYSIS

By analyzing the combined failure mechanism taking into account all already given values of random variables in the limit function, different indexes of reliability are obtained for different relationships between rigidity of beams and columns. Based on the results from Fig. 3 it can be seen that increasing the rigidity relationship between beams and columns increases reliability index

By changing the dimension for only 5cm, reliability index increases on average around 8%. This increase in the beginning is a lot, because the dimension of 25cm reliability index is 2.83 while dimension of 30 cm index gains value 3.25 which is 15% increase. This percentage considerably decreases for dimension of 40cm and is 7.4%.
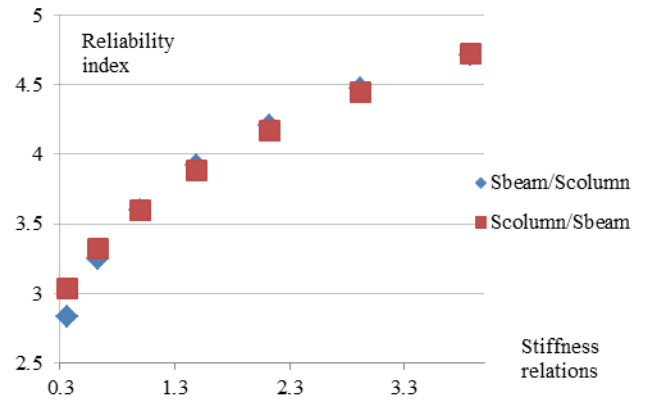

Fig. 3.   Influence on stiffnes relations between beam and column on reliablity index

Reliability indexes are approximately the same, if the ratio of rigidity between columns and beams is equal to one or greater than one. If their ratio is less than one, reliability index is lower for lower rigidity of the beam.

Reliability index from Table 4 is obtained when the ratio of rigidity between beams and columns is the same and it approximately meets standardized reliability index of 3.8 according to [7] [8] [9]. The probability of failure according to FORM method is $1.587 \cdot 10^{-4}$, while similar value is obtained from the SORM to, $1.590 \cdot 10^{-4}$.

TABLE IV.   RELIABILITY OF SWAY-BEAM MECHANISM

Mechanism	Reliability index β for return period of 50 years according to FORM Method	Probability of failure P_f for return period of 50 years according to FORM method	Reliability index β for return period of 50 years according to SORM Method	Probability of failure P_f for return period of 50 years according to SORM method
Combined	3.60	$1.587*10^{-4}$	3.60	$1.590*10^{-4}$

Data on the reliability indexes from Table IV and Fig. 3 are got in the case when the surface of reinforcement in the columns and beams is unchanged and is 4Rϕ19 (11.34 cm^2).

On Fig.4 is displayed a variation of the reliability index depending on the different reinforcement in the columns and beams while the geometry of columns and beams remains unchangeable and all the dimensions of the cross section are 35 cm according to Table 3. The data shown on Fig. 4 are obtained by assuming that the reinforcement in the beams is simultaneously increased in the field and over the support.

Based on the results from Fig. 4 it is clear that with the increase of reinforcement surface in the beams the reliability index is significantly increased. For the average increase of the reinforcement in beams of 3.37cm2 the reliability index is in average increases for 21.5 %, while the maximum increase of reliability index is 27.2% for the increase of reinforcement from 4Rφ16 to 4Rφ19. With increase of the reinforcement of the columns the reliability index is slightly increased. The biggest increase of this index is 3.61% when the reinforcement of the columns is increased from 4Rφ19 to 4Rφ22, while the average increase is 2.76%.
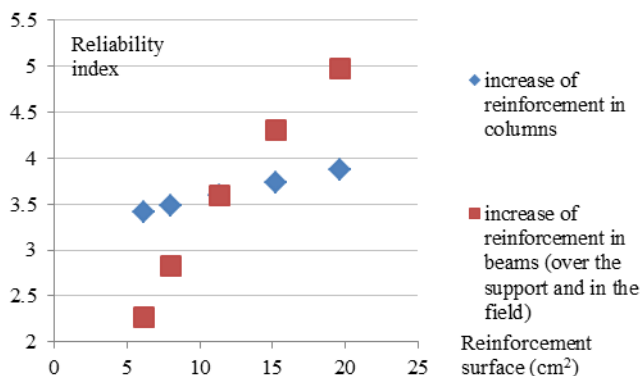

Fig. 4.   The impact of increasing reinforcement on reliability index

On Fig.5 is shown the influence of the increase of reinforcement in the field of beam and over support, while the reinforcement in the columns stays unchangeable and is 11.34 cm2.
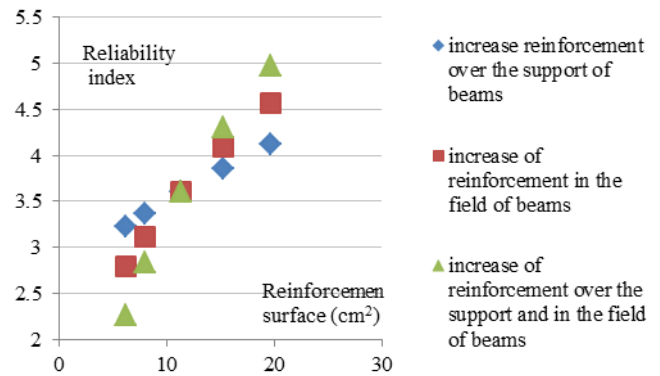

Fig. 5.   The imact of increasing reinforcement in beams on reliability index

With the increase of reinforcement in the field it is clear that it also comes to even bigger increase of reliability in relation to the reinforcement increase in the field which is logical if we look back on the limit function for combined mechanism (21).

V.   CONCLUSION

Based on all the diagrams Fig.3, Fig.4, and Fig.5 it is clear that the influence of reinforcement is much bigger than geometrical characteristic of the frame. The highest reliability index is obtained by increasing the reinforcement in the beams and in particular by increasing reinforcement in the field of beam which is logical if we look at the limit function (21) for combined mechanism.

Creating plastic hinges in the columns of the frame we try to avoid, that is why the combined mechanism of the frame is most desirable, because there is the smallest probability of creating brittle fracture. Plastic joints that occur in beams in areas of maximum impact should be as ductile as possible and that they can deform longer without brittle fracture.

Other authors, such as [10] discussed the reliability of reinforced concrete beams dimensioned according to ACI code taking into account the effects of bending, shear and torsion, as well as the influence of resistance and load on the reliability index. Unlike [10] this paper is based only on determining the probability of fracture, taking into account the presumption that breakage will not occur due to fracture in concrete, it will occur in the reinforcement, which corresponds to ductile behavior of concrete structures. The paper [11] shows the reliability analysis of very high beam dimensioned also with the ACI code for several limit functions of fracture and comparative analysis of the results obtained by numerical and laboratory was conducted. Many researchers [12]. [13] analyzed the reliability of reinforced concrete frame due to the effect of horizontal seismic forces while this work is based on the influence of wind forces.

This paper had a task to show how the stiffness and bearing capacity of some constructive elements of the frame have an impact on the creation of the desired fracture mechanism. The basic assumption of these results is that it is not taken into the

account the influence to shearing or torsion but only to bending as the dominant influence.

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